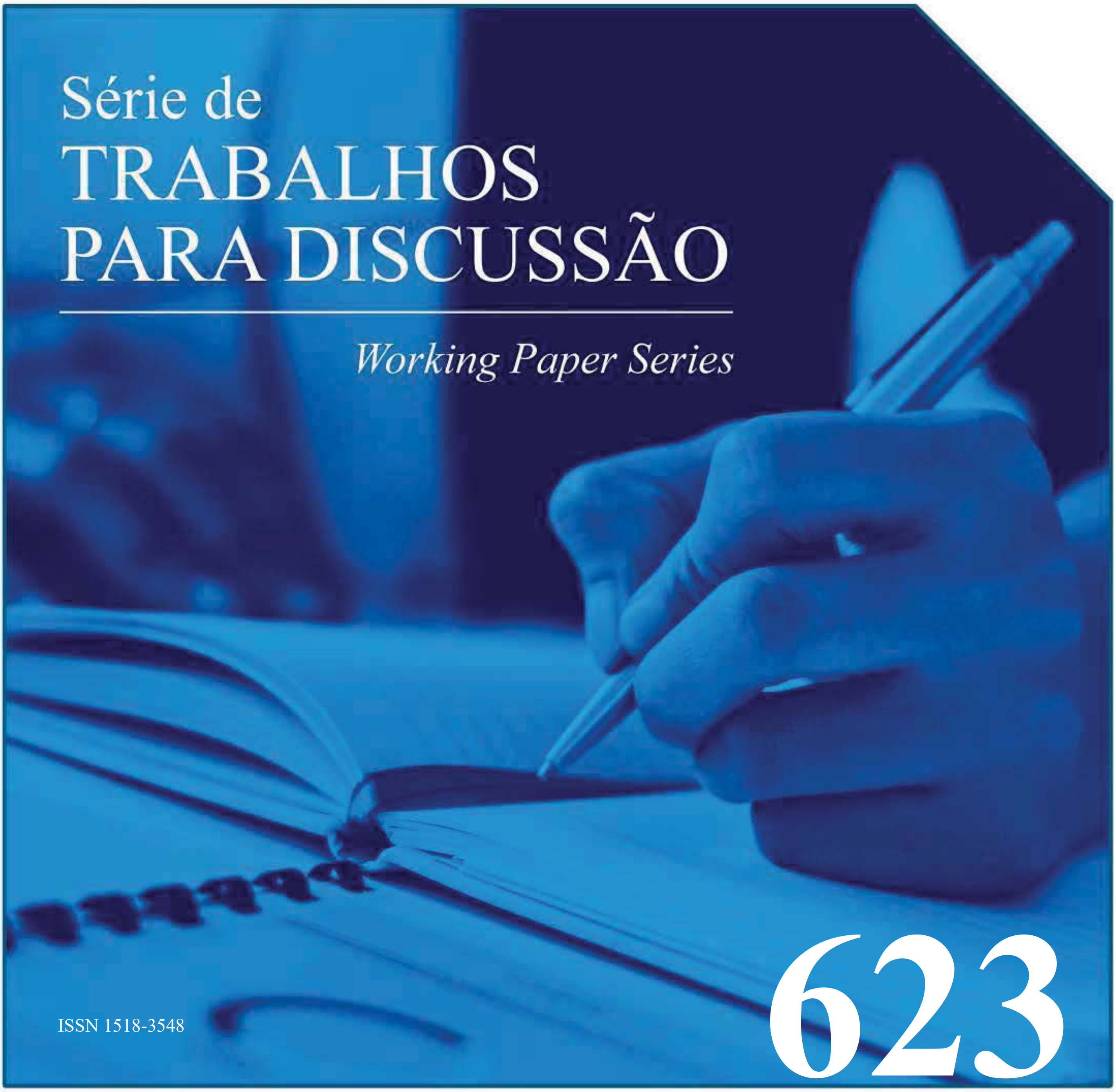

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Optimal Inflation Targeting

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Non-Technical Summary

In recent years, inflation has become a major economic concern, especially following the COVID-19 pandemic and global geopolitical tensions. Rising prices for essential goods like food and energy, along with supply chain disruptions, have pushed inflation to levels not seen in decades. Central banks worldwide have responded with significant interest rate hikes to stabilize inflation and inflation expectations, seeking to restore economic balance. This paper explores the concept of optimal inflation targeting — the best inflation target level, and the optimal response to deviations from the target across the time. It examines how inflation impacts households' well-being and how monetary policy can be designed to respond effectively to inflation and other economic shocks.

A key aspect of this study is the way firms adjust their prices. While standard economic models assume businesses set prices based on future expectations, this paper considers a more realistic approach, where some firms look at past inflation when resetting their prices. This behavior can have significant effects on how inflation spreads through an economy, how it evolves and how it stubbornly sticks. The paper suggests central banks should carefully operate inflation targets to avoid excessive economic volatility, and especially, negative impacts on households' welfare.

The study also explores how different inflation targets affect economic welfare. It finds inflation creates costs for businesses and households, making life more unpredictable, and reducing economic efficiency. Using simulations, the paper shows a zero inflation target is the best monetary policy target. Additionally, the paper shows increases in inflation targets call for increases in monetary policy's response to inflation deviation from the target, reinforcing two concepts: i) the importance of credibility in monetary policy; ii) the growing welfare costs of inflation for households. Finally, when addressing conjuncture shocks, optimal monetary policy never reacts to output or production gaps.

Ultimately, this research highlights the importance of carefully structured and planned monetary policy to keep inflation under control. It argues that the optimal inflation target, that maximizes households' welfare, is zero. Hence, higher levels of inflation produce unnecessary losses in households' welfare and economic efficiency.

Sumário Não Técnico

Nos últimos anos, a inflação se tornou uma grande preocupação econômica, especialmente após a pandemia de COVID-19 e as tensões geopolíticas globais. O aumento dos preços de bens essenciais, como alimentos e energia, juntamente com as interrupções nas cadeias de suprimentos, impulsionou a inflação para níveis não vistos em décadas. Bancos centrais ao redor do mundo responderam com aumentos significativos nas taxas de juros para estabilizar a inflação e as expectativas de inflação, buscando restaurar o equilíbrio econômico. Este artigo explora o conceito de *optimal inflation targeting* — o melhor nível de meta de inflação e a resposta ideal às suas flutuações ao longo do tempo. Examina como a inflação afeta o bem-estar das famílias e como a política monetária pode ser projetada para responder eficazmente à inflação e a outros choques econômicos.

Um aspecto-chave deste estudo é a forma como as empresas ajustam seus preços. Enquanto os modelos econômicos padrão assumem que as empresas definem seus preços com base em expectativas futuras, este artigo considera uma abordagem mais realista, onde algumas empresas olham para a inflação passada ao reajustar seus preços. Esse comportamento pode ter efeitos significativos na disseminação da inflação na economia, na sua evolução e na sua persistência. O artigo sugere que os bancos centrais devem operar cuidadosamente seus regimes de metas de inflação, de forma a evitar excessos de volatilidade econômica e, especialmente, impactos negativos no bem-estar das famílias.

O estudo também explora como diferentes metas de inflação afetam o bem-estar econômico. Ele encontra custos de inflação para empresas e famílias, tornando a vida mais imprevisível e reduzindo a eficiência econômica. Utilizando simulações, o artigo mostra que uma meta de inflação zero é a melhor estratégia de política monetária. Além disso, demonstra que aumentos na meta de inflação exigem respostas mais intensas da política monetária às variações da inflação em relação à meta, reforçando dois conceitos: i) a importância da credibilidade na política monetária; ii) os custos crescentes da inflação para o bem-estar das famílias. Por fim, ao lidar com choques conjunturais, a política monetária ótima nunca reage a desvios do produto ou da produção.

Em última análise, esta pesquisa destaca a importância de uma política monetária bem estruturada e planejada para manter a inflação sob controle. Portanto, níveis mais altos de inflação produzem perdas desnecessárias no bem-estar das famílias e na eficiência econômica.

Optimal Inflation Targeting

Pedro Henrique Alves Pereira*

Abstract

This paper examines optimal inflation targeting, determining the best inflation target and policy response using a DSGE model with adaptive price-setting firms, in the form of a hybrid Phillips Curve. The results indicate a zero inflation target maximizes household welfare, as higher inflation increases volatility and economic inefficiency. As inflation targets rise, monetary policy must respond more aggressively to shocks and deviations from the target. Notably, the optimal policy does not react to output gaps. These findings highlight the importance of credible and well-structured monetary policies in ensuring macroeconomic stability and the growing costs of inflation (in terms of households' welfare).

Keywords: Monetary Policy, Optimal Policy, Macroeconomics, Inflation Targeting, Central Bank, Interest Rates; Business Fluctuations; Cycles; Policy Making, Policy Rules.

JEL Classification: E300; E320; E500; E520; E600; E610.

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1 Introduction

After The Great Moderation, the period that followed the Great Financial Crises of 2008, when inflation and inflation expectations were kept well anchored, the recovery from the COVID-19 pandemic has been accompanied by fast increases in inflation, levels not seen in decades. Perhaps even more relevant, long-run inflation expectations had been de-anchoring. This triggered significant interest rate hikes to bring back long-run inflation expectations. The inflation watershed on monetary policy was triggered by the sharp rise in food and energy prices following the Russian invasion of Ukraine. The rise in geopolitical risks had also triggered logistic and manufacturing resetting across the world supply, labeled as ”nearshoring” and ”friendshoring”, weighting on firms’ costs and consequently on inflation. Last but not least, the monetary and fiscal stimulus released during the pandemic have also weighted in inflation and inflation expectations.

This paper studies the case of optimal inflation target with a small departure from standard pure forward looking models. I study the impacts of inflation on economic welfare and dynamics, and what is the optimal inflation targeting. Optimal inflation targeting involves the optimal inflation target rate and the optimal response to shocks (i.e., the optimal parameters on the Taylor Rule). The departure from pure forward looking models is expressed by the adaptive behaviour that firms might take, sometimes, when resetting their prices. Such as in Galí and Gertler [1999], some firms reset their prices by the previous period’s new price, adjusted by the previous period’s inflation. Under this framework, which is possibly more realistic than standard forward looking models, this article seeks to answer what would be the optimal inflation target, and how would be the optimal monetary policy, under different inflation targets. In other words, it seeks to answer what is the optimal inflation targeting.

According to King [2024] Inflation Targets were successful because of procedures and institutions for setting monetary policy in a transparent and accountable fashion: ”constrained discretion”. Yet, new macroeconomic consensus started to develop macroeconomic models that abstract from money. Two fundamental results of these models were that present inflation deviation is explained by present deviations of real variables and long-run inflation expectations. To close the model, long-run inflation expectations were always determined by the inflation target. King [2024] argues the consequence is that Milton Friedman’s old dictum was rewritten as: inflation is always and everywhere a transitory phenomenon.

Palma and Portugal [2014] estimate the preferences of the Central Bank

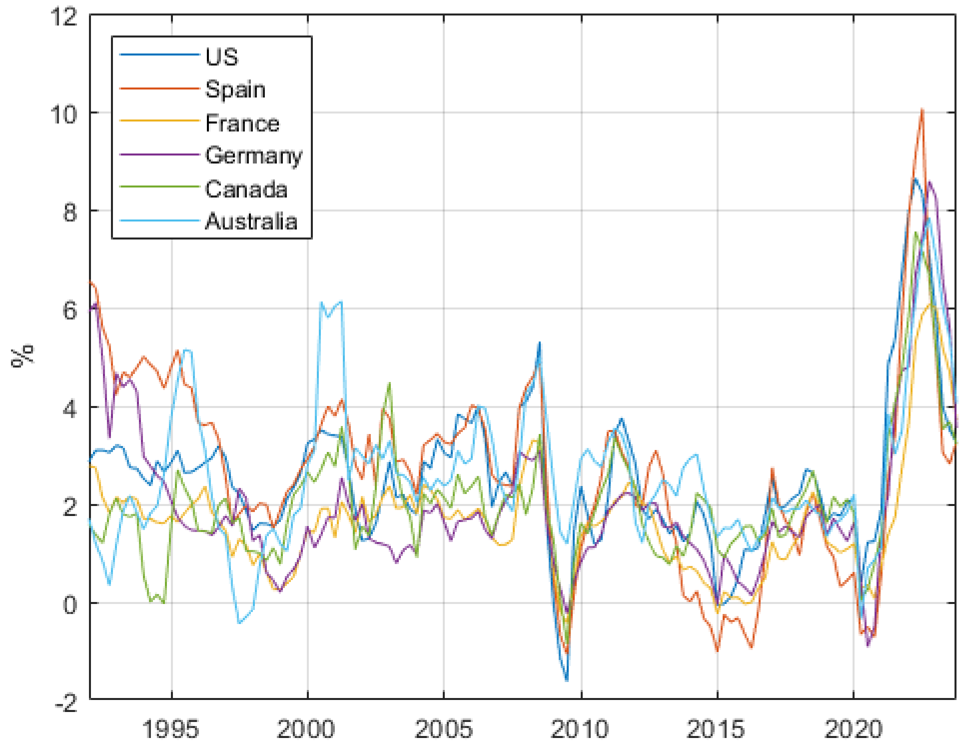

Figure 1: Consumer Price Index (CPI): Growth rate same period previous year (QoQ).

of Brazil (BCB) after the inflation target regime was implemented, with a small open economy DSGE model. To set the monetary policy the Central Bank minimizes a loss function, taking into account the deviation of inflation from its target, output stabilization, interest rate smoothing, and the exchange rate dynamics. The authors follow Galí and Monacelli [2005] and Justiniano and Preston [2010]. The paper is clear and quite standard, and sheds light on the dynamic of monetary policy in Brazil, favoring the debate and improvements of the monetary policy framework. The results show the BCB's main concern during this period was inflation stabilization, followed by interest rate smoothing, exchange rate stabilization, and output stabilization. The large value of the exchange rate smoothing parameter suggests the presence of a "fear of floating". The authors argue that a more passive stance concerning foreign exchange floating would improve the inflation target regime in Brazil.

This article has some similarities with Palma and Portugal [2014]. Like Palma and Portugal [2014], the behavior of monetary authority is studied, while Palma and Portugal [2014] estimate the real behavior, and here the theoretical optimal behavior or policy is considered, in the context of optimal simple rules. Households in Palma and Portugal [2014] present external habits, but apart from that, the utility function is quite similar. Some differences are worthy to note. The economy here presented is closed while Palma and Portugal [2014] work with a small open economy; such as has been common in the literature; Palma and Portugal [2014] abstract from money, while here it does not. The presence of money facilitates the emergence of inflation costs into DSGEs.

Costa and Carrasco-Gutierrez [2015] study whether there is a fraction of non-optimizer consumers, who do not optimize consumption across lifetime, consuming all the disposable income at every period, following the seminal works of Campbell and Mankiw [1989] and Campbell and Mankiw [1990]. The authors found evidence that there is a significantly large portion of households behaving in a non-optimal fashion, in sharp contrast with Friedman [1957] permanent income hypothesis.

In this article I also follow a non-optimizing behavior, but instead of regarding household consumption it is related to firms price setting. Following Galí and Gertler [1999] I adopt Calvo prices and let a random share of firms that reset their prices, by adjusting their prices by the previous new prices adjusted by inflation. The complementary share of firms that are allowed to reset their price behave optimally, in a forward looking fashion. There are many reasons why some firms might simply behave as backward looking price setters, such as: i) persistent high and persistent volatile inflation; ii) higher levels of indexation, especially regarding specific firms' operation; iii) lack of credibility in monetary policy.

Andrade et al. [2018] study how changes in steady-state real interest rate affect the optimal inflation target, using an estimated New Keynesian DSGE model that incorporates the effective lower bound. Interestingly, the authors found a negative slope (-0.9) between the real interest rate and the optimal inflation target. Andrade et al. [2018] also found optimal inflation targets for estimated models of the US and EU at 2 percent and 1.5 percent, respectively, when there is no uncertainty regarding model parameters by the policymaker.

I found optimal inflation targets much slower than the ones found by Andrade et al. [2018]. This may be due to the differences in the models. Andrade et al. [2018] work with a non-stationary open economy model estimated DSGE model, while I work with a stationary, calibrated to an emerging market economy, to begin with. Maybe even more important for the results are the models themselves, where Andrade et al. [2018] works with a more standard benchmark model without money, while I allow for money and bank deposits explicitly, such as in Christiano et al. [2005], but also adding a cash-in-advance (CIA) constraint.

Alvarez and Dizioli [2023] study the cost of reining in inflation in terms of output and employment, in the context of a post-pandemic world. Inflation had made itself felt across the globe, reaching levels not seen in decades. The econometric models suggested the cost of controlling inflation would be high, implied by the slope of the Phillips Curve. Yet these econometric models, calibrated and estimated at the period of great moderation [1] when inflation was controlled and inflation expectations were well-anchored, were unable to forecast and explain the rise of inflation and its persistence. The authors forcefully argue that the inflation cost could be actually low, considering that expectations in reality should not be fully rational but limited rational instead. Alvarez and Dizioli [2023] made use of the learning techniques to address the post-pandemic economic developments. In the model households estimate leading variables with statistical models with a smaller set of variables. Households learn from mistakes and use forecast errors to update parameter values with a Kalman filter. The authors work with a canonical New-Keynesian model, with sticky prices and wages. The learning technique implies a time-dependent backward looking economic system.

This work has a number of similarities with Alvarez and Dizioli [2023]. Like Alvarez and Dizioli [2023] I also depart from rational expectations,

[1] the period after the Great Financial Crisis

but instead of the limited rationality technique, I followed Galí and Gertler [1999] in the more simple technique of adaptive expectations. But different from Alvarez and Dizioli [2023] I make use of a less parsimonious model, in order to account for the cost of long-run inflation. Since wages are flexible the degree of nominal stickiness is lower in our model when compared to Alvarez and Dizioli [2023]'s model.

Araujo et al. [2023] study the relation between optimal monetary policy and high debt government. The study adds to the literature on the fiscal limits of monetary policy and the interdependence between fiscal discipline and price stability, which is well addressed in the literature. High government debt, (the so-called Fiscal Fragility Zone (FFZ)) implicates a high cost to meet inflation target, and the benevolent government uses higher inflation to generate fiscal revenue, partially improving the fiscal situation. Agents form expectations rationally, and for low debt levels, the inflation target is always met. For high debt levels emerge the trade-off studied. In this case, and for a low inflation target, agents expect the inflation target not to be met. Within the FFZ, the policymaker is subject to confidence crises, so the optimal policy may be to gradually reduce the public debt, mitigating confidence crises and reinforcing the inflation-target regime. The role of the inflation target is critical: higher targets avoid confidence crises and facilitate the exit from the FFZ, mitigating the risk of missing the inflation target.

Like Araujo et al. [2023] this paper also seeks to understand the costs of inflation, the role of expectations and the optimal monetary policy. But differently from the authors I abstract from the fiscal sector. Unfortunately, the abstraction is not costless, since some emerging markets, such as Brazil and Argentina, present almost secular, chronic fiscal issues. Also, the fiscal unbalances could (theoretically and probably) promote an inflation dynamic a bit particular to these economies, restricting the results of this work. Hence, to avoid the less general results and also to avoid the complications of tax distortions, I abstract from the fiscal sector. Araujo et al. [2023] adopt the more standard rational expectations case, whereas in the case of price formation, I depart from it, following Galí and Gertler [1999]. The authors conclude a low inflation target may not be welfare-improving if it raises doubts over the commitment of the policymaker to provide the necessary fiscal backing for delivering the target. But it could also symmetrically apply to a higher reset inflation target. If agents believe the higher inflation target will be accompanied by higher fiscal expansion, as some fiscal authorities many times directly imply, the fiscal revenues from inflation are absorbed in new expansion the economy is kept in the FFZ, or maybe even in a more fragile point in it.

This chapter contributes with the literature with an assessment of inflation costs, impacts on welfare and economy dynamics. Also, what is the optimal inflation target and what is the optimal monetary policy, expressed in the Taylor Rule in the context of simple rules. The next section presents the model applied. The calibration section presents the calibration and steady-state equilibrium. The results section presents the results and the last section concludes.

2 The Model

The model is built in the lines of Cooley and Hansen [1989], Fuerst [1992] and Galí and Gertler [1999]. It is a fairly simple closed economy business cycle model, with nominal stickiness; households maximize welfare and firms maximize profits. Like Fuerst [1992] households are required to hold money to buy consumption goods (and services) and to make deposits in banks.

To allow for helicopter drop of money (such as in Cooley and Hansen [1989], represented here by $x_t^f M_{t-1}$ [2]) I modify the Fuerst [1992] CIA constraint, imposing one lag [3] for household money held to be exchanged into consumption[4]. This implies a share of money issuance is not directly controlled by the monetary authority. One can understand this structure as institutional arrangements regarding the issuance of money that are not under the control of the monetary authority, in the form of legal or even constitutional demands regarding, direct or indirect, money financing of certain public issues. A historically well-known theme is to cover a share of fiscal deficits with money issuance. Another more subtle form is the monetary authority to remunerate the government for its account positions,

[2]where M_{t-1} is the aggregate money stock from the previous period, and previous period x_t^f is an exogenous stationary auto-regressive process.

[3]such as in Cooley and Hansen [1989].

[4]The additional lag on money balances is less common in the literature, with many authors allowing present money to finance present consumption, such as Christiano [1991], Carlstrom and Fuerst [1995], Christiano et al. [2005] or Schmitt-Grohé and Uribe [2007]. In this case, I would follow the standard here, it would make labor hours cheaper. Households could arbitrage between deposits and money held. For instance, if households make abnormally high deposits to fund labor, they receive this higher than normal income from deposits and labor from banks and firms. And these extra resources could be held in money to consume, everything inside the discrete time period. The model would work at least as well as it is working otherwise, but there could exist this free lunch.

which is simply legal money issuance[5]. Price stickiness is imposed following Galí and Gertler [1999].

2.1 Households

As usual households maximize utility, which is a function of discounted consumption and labor across time. The objective function is subject to two constraints: the CIA constraint and the budget constraint (or also known as the law-of-motion).

Like Lucas [1980], each household is composed of two persons, the worker and the shopper. When the period starts, the worker goes to the firm [6], while the shopper goes shopping for different products than the ones produced by his worker pair, with money balances held from the previous period. At the beginning of the period, the households decide how much to consume and how much to deposit in banks, limited to money balances held in the previous period. The household labor pair comes back home with income from the family factors. Afterwards, they decide on investment and money hold to be used in deposits and consumption goods at the next period.

At every period the representative household seeks to solve the following problem:

$$\max_{\{c_t,h_t,m_t,k_{t+1},n_t\}} \quad E_t \sum_{t=0}^{\infty} \beta^t u(c_t,h_t) \tag{1}$$

subject to the CIA constraint:

$$P_t C_t \leq m_{t-1} + x_t^f M_{t-1} - N_t, \tag{2}$$

and to the budget constraint:

$$P_t k_{t+1} + m_t = P_t w_t h_t + P_t(r_t^k + 1 - \delta)k_t + P_t d_t + R_t^n n_t, \tag{3}$$

I work with a basic utility function: $u(c_t,h_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma} - \theta\frac{h_t^{1+\varphi}}{1+\varphi}$. Let μ_t and λ_t denote the Lagrange multipliers of the CIA constraint and the budget constraint. The first-order conditions are:

$$c_t^{-\sigma} = \mu_t, \tag{4}$$

[5]Note that this monetary issuance could also be related market sources, such as an increase in the velocity of money that increases the quantity of money.

[6]Different from Lucas [1980], where the worker goes to the tree to collect fruits.

$$\lambda_t = \theta \frac{h_t^{\varphi}}{w_t}, \tag{5}$$

$$\beta E_t \frac{\mu_{t+1}}{P_{t+1}} = \frac{\lambda_t}{P_t}, \tag{6}$$

$$\lambda_t = \beta E_t \lambda_{t+1}(r_{t+1}^k + 1 - \delta), \tag{7}$$

$$\mu_t = \lambda_t R_t^n. \tag{8}$$

To better understand what is happening here, let's consider Hansen [1985]'s RBC model. For the baseline model at Hansen [1985] we have the Euler equation for capital as: $u'_{c_t} = \beta E_t u'_{c_{t+1}}(r_{t+1}^k + 1 - \delta)$, which means the marginal utility of present consumption should equal the expected time discounted marginal utility of next period consumption times the expected net gross interest rate of capital [7]. Here with money, CIA and inflation, one could find: $E_t \frac{u'_{c_{t+1}}}{\Pi_{t+1}} = \beta E_t \frac{u'_{c_{t+2}}}{\Pi_{t+2}}(r_{t+1}^k + 1 - \delta)$. There is no money in Hansen's baseline RBC, so households could consume directly the extra income. As the CIA imposes, the extra resource should be held in money for one period, subject to inflation. Only after this the household can go shopping with the money hold. So the expected next period marginal utility of consumption discounted by inflation equals the two periods ahead discounted marginal utility of consumption times the expected gross rate of return of capital. One could also see the reasons as the marginal utility of holding money at some period; in any case, the relations are reminiscent of the RBC model.

Next, consider the Euler equation for labor. In Hansen [1985] one could find: $u'_{c_t} = -u'_{h_t}/w_t$, or the marginal utility of consumption equals the negative of the marginal utility of labor divided by the (real) wage rate. Again, as labor income could not be turned into consumption directly the labor costs should be related to expected consumption; for the same reason, the wage rate should be discounted by the next period inflation. Therefore, one could substitute the FOC and find the Euler condition for labor: $E_t u'_{c_{t+1}} = -\frac{u'_{h_t}}{w_t/\Pi_{t+1}}$.

[7] If the return of capital is expected to be higher in the future, it is worth to decrease consumption today (which increases u'_{c_t}), investing the extra resources in capital to take advantage of the expected good times.

2.2 Banks

Likewise Christiano et al. [2005], banks operate competitively, accepting deposits from households and lending them to firms to fund payrolls. So, the nominal payroll is funded by deposits plus monetary issuance by the central bank through open market operations:

$$N_t + x_t^m M_{t-1} = P_t w_t h_t, \tag{9}$$

where x_t^m denotes the money rate issuance by the monetary authority. As banks operate in competitive markets, banks' costs equal income, as below:

$$R_t^n N_t = R_t^l P_t w_t h_t, \tag{10}$$

2.3 Firms

The productive sector follows a fairly standard and simple structure. Intermediate Good Firms operate in a monopolistic competition, facing a Calvo prices fashion. At every period firms rent labor and capital to produce output. These firms sell the intermediate goods to Final Good Firms so that they can produce output. Final Good Firms, operating in perfect competition, sell output to households, that can be consumed or transformed into capital.

2.3.1 Final Goods Firms

Final goods firms operate in competitive markets, buying intermediate goods and combining them to produce a final good with the following technology: $Y_t = (\int_0^1 Y_t(j)^{(\psi-1)/\psi}\, dj)^{\psi/(\psi-1)}$. Hence the final goods firms must solve the following problem:

$$\begin{aligned} &\max_{Y_t(j)} \Big\{ P_t Y_t - \int_0^1 P_t(j) Y_t(j)\, dj \Big\} \\ s.t. : Y_t &= \left(\int_0^1 Y_t(j)^{(\psi-1)/\psi}\, dj \right)^{\psi/(\psi-1)} \end{aligned} \tag{11}$$

Solving the final goods firms problem we get the well know demand for any "j" intermediate good: $Y_t(j) = Y_t(P_t/P_t^j)^{\psi}$.

2.3.2 Intermediate Goods Firms

Intermediate goods firms produce according to the same technology across them, the Cobb-Douglas production function: $Y_t(j) = Z_t K_t(j)^{\alpha} H_t(j)^{1-\alpha}$, and sell their output to final good firms. Intermediate goods face two problems. Cost minimization, at every period, and inter-temporal price optimization, when they are randomly called to set their prices.

A firm cost minimization can be expressed as:

$$\min_{K_t(j),H_t(j)} \left\{ r_t^k K_t(j) + R_t^l w_t H_t(j) \right\} \tag{12}$$

$$\text{s.t.:} Y_t(j) = Y_t \left(\frac{P_t}{P_t(j)} \right)^{\psi} = Z_t K_t(j)^{\alpha} H_t(j)^{1-\alpha} \tag{13}$$

The first-order conditions for the above problem[8] are:

$$r_t^k = \alpha mc_t Y_t / K_t(j) \tag{14}$$

$$R_t^l w_t = (1-\alpha) mc_t Y_t / H_t(j) \tag{15}$$

where mc_t denotes the Lagrange multiplier of the constraint, but also denotes firm's real marginal cost.

2.3.3 Forward Looking Intermediate Goods Firms

The price optimization of intermediate good firms can be expressed as follow:

$$\underset{P_t^f}{max} \left\{ E_t \sum_{i=0}^{\infty} (\beta\rho)^i \frac{u'(c_{t+i})}{u'(c_t)} \left[P_t^f Y_{t+i}(j) - P_{t+i} mc_{t+i} Y_{t+i}(j) \right] \right\}. \tag{16}$$

These firms observe the demand function for their products, as a function of aggregate demand and the relative price. Substituting the demand function for $Y_t(j)$, one can find the Euler equation for forward looking firms' price setting as:

$$P_t^f = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i u'(c_{t+i}) mc_{t+i} Y_{t+i} P_{t+i}^{\psi}}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i u'(c_{t+i}) Y_{t+i} P_{t+i}^{\psi-1}} \tag{17}$$

[8]Substituting back to the Cobb Douglas, one can find the optimal demand for factors. Substituting this optimal demand in the objective function, one can find real total costs for firm "j", as a function of the production level of the firm, and market variables, such as: $RC(j) = F(Y_t(j), w_t, R_t^l, r_t, Z_t)$.

2.3.4 Backward Looking Intermediate Goods Firms

Like Galí and Gertler [1999], Backward Looking Intermediate Goods Firms simply reset their prices by previous period inflation:

$$P_t^b(j) = P_{t-1}^* \frac{P_{t-1}}{P_{t-2}} \tag{18}$$

2.4 A Structural Phillips Curve

In this subsection we follow closely Galí and Gertler [1999], with the minor modification of allowing the sticky price to be adjusted by the inflation target. The price index is $P_t^{1-\psi} = \int_0^1 P_t(j)^{1-\psi} dj$. Since a ρ share of firms reset their prices by target inflation, and the complementary share reset their prices in the form of standard forward-looking optimization [9], we shall have: $P_t^{1-\psi} = \int_0^\rho [\overline{\Pi} P_{t-1}(j)]^{1-\psi} dj + \int_\rho^1 P_t^{*1-\psi} dj$. Hence we shall have:

$$P_t^{1-\psi} = \rho(\overline{\Pi} P_{t-1})^{1-\psi} + (1-\rho) P_t^{*1-\psi}. \tag{19}$$

New prices are reset by backward and forward looking firms:

$$P_t^{*1-\psi} = \omega P_t^{b\,1-\psi} + (1-\omega) P_t^{f\,1-\psi} \tag{20}$$

Log-linearizing the equations (10)-(13), after a bit of algebra one can find a hybrid Phillips curve:

$$\Big[\rho + \omega\Big(1 - \rho(1-\beta)\Big)\Big]\tilde{\pi}_t = \beta\rho\tilde{\pi}_{t+1} + \omega\tilde{\pi}_{t-1} + (1-\rho)(1-\omega)(1-\beta\rho)\tilde{mc}_t \tag{21}$$

2.5 Market Clear

The capital dynamics is:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{22}$$

Aggregate demand and aggregate supply are described by:

$$Y = C_t + I_t, \tag{23}$$

$$Y_t = \frac{Z_t K_t^\alpha H_t^{1-\alpha}}{V P_t}, \tag{24}$$

[9] See 17

where VP_t denotes the price variation function. As commonly, from the demand for differentiated goods we can find the variation in prices function:

$$VP_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\psi} dj. \tag{25}$$

Money dynamics is a function of exogenous issuance from the fiscal authority given to households (Helicopter drop) and the endogenous injection in the financial markets by the central bank. In the CIA constraint there is a money issuance (like a lump-sum) for households, at a rate of x_t^f over the previous stock of money, then the new money issued and given to households equals $x_t^f M_{t-1}$. The monetary policy followed by the Taylor Rule also implies a money injection/withdrawal, at the rate of x_t^m. Similarly, the new money issued by the central bank that goes into the system at time t is $x_t^m M_{t-1}$. Therefore, the money dynamics is:

$$M_t = (1 + x_t^f + x_t^m) M_{t-1}. \tag{26}$$

Natural output is measured by the long-run output level adjusted by the current state-of-arts, or productivity level. In other words, natural output is simply equilibrium steady-state output adjusted by the current productivity level:

$$Y_t^n = Z_t \bar{K}^\alpha \bar{H}^{1-\alpha}. \tag{27}$$

As usual, output gap is the simple relation between output and natural output:

$$X_t = \frac{Y_t}{Y_t^n}. \tag{28}$$

2.6 Monetary Policy

The monetary authority observes and targets the interest rate that is being practised in the loan markets. The central bank follows a Taylor-type rule, as below:

$$R_t^l = \rho_r R_{t-1}^l + (1 - \rho_r)\left[\zeta_\pi\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \zeta_x\left(X_t\right)\right] + \epsilon_t^r, \tag{29}$$

where ϵ_t^r is a white-noise process.

2.7 Shocks

The TFP process is:

$$Z_t = (1-\rho_z)\bar{Z} + \rho_z Z_{t-1} + \epsilon_t^z \quad (30)$$

In a similar way, the exogenous fiscal issuance rate process is:

$$x_t^f = (1-\rho_{xf})\bar{x^f} + \rho_{xf} x_{t-1}^f + \epsilon_t^{xf} \quad (31)$$

3 Calibration

I seek to adopt a more or less generic calibration for the model, with minor deviations. At first look one can believe the inflation coefficient in the Taylor rule does not abide by the Taylor Principle. This may not be the case, because credit operations are realized in an intraperiod fashion; in other words, the interest rate target R_t^L is a real interest rate, so any positive values for ζ_π in theory, abides the Taylor Principle. I arbitrarily pick $\bar{x^f}$ so that the annual money growth that goes to finance fiscal budget[10] would be about 4%. This value implies *seigniorage* levels close to many real economies. Different values for $\bar{x^f}$ do not change the main results of this study and the model performs, *mutatis mutandis*, in a likewise fashion[11]. For price stability, positive values for $\bar{x^f}$ result in higher equilibrium interest rates, as the theory stands and evidence suggests. In general, the Monetary Authority shall keep higher interest rates, drawing resources from the economy to stabilize inflation.

The parameters chosen imply the non-stochastic steady state presented in Table 2. Note from Table 1 that money issuance rate by fiscal authority is positive, and in the baseline the monetary authority seeks zero inflation in the long run. This commitment must imply here a zero long-run money growth rate. Therefore, the positive money issuance by the fiscal authority implies a negative rate with the same magnitude. For this, the monetary authority increases the interest rate (R^l) above the natural level with zero money issuance, which equals the deposit rate (R^n). What happens is that the monetary authority dawns the new money coming into the economy, although by a different channel. The new money is ready to be exchanged for consumption goods or deposits, while the sterilization conducted by the monetary authority is exclusively on deposits, or the financial system.

[10] Which in this case is equal to long-run money growth rate.

[11] Obviously, for a first order approximation, one should pick $\bar{x^f} \neq 0$: otherwise these shocks would disappear, for a first order approximation.

Description	Parameter	Value
Capital Share	α	0.3
Inverse of Risk Aversion	σ	1.6
Inverse of Labor Supply Elasticity	φ	.9
Labor Aversion	θ	5
Discount Factor	β	0.99
Depreciation Rate	δ	0.025
Elasticity of Substitution - Intermediate Goods	ψ	11
Calvo Parameter - Prices	ρ	0.72
Interest Rate Smoothing	ϕ_r	0.825
Policy Rate Response to Inflation	ζ_π	0.5
Policy Rate Response to Output-gap	ζ_x	0.125
Backward Looking Ratio	ω	0.5
TFP persistence	ρ_z	0.95
Money Issuance steady state rate by Fiscal Authority	$\bar{x}^f$	0.01

Table 1: Baseline Parameters and Target Values for Variables.

4 Results

The Impulse Response Functions (IRFs) are presented in Appendix. Note the model presents hump-shaped responses to the three shocks on TFP, Money Issuance and Monetary Shock [12]. The TFP shock implies a policy rate cut, due to inflation decreasing below the target. Regarding Monetary Policy Shock, apart from consumption response, the responses are very good. Consumption increases a bit right after the shock, because prices unexpectedly decrease, and this increases real balances used in consumption at the period of the shock.

To assess welfare issues, I run a Monte-Carlo experiment, with one hundred thousand periods, repeated fifty times. In this section I present results regarding the optimal inflation target and the optimal monetary policy that minimizes a loss-function.

4.1 The Optimal Inflation Target

I follow Lucas [1992] and Schmitt-Grohé and Uribe [2001] to measure the cost of the business cycle. I find the fraction of non-stochastic lifetime con-

[12] I omit the statistic simulations, but they are not disappointing.

Description	Variable	Value
Output	$\overline{Y}$	0.8912
Consumption	$\overline{C}$	0.7181
Labor Hours	$\overline{H}$	0.3701
Aggregate Capital	$\overline{K}$	6.9243
Wage	$\overline{W/P}$	1.4826
Rentals rate	$\overline{r^k}$	0.0351
Investment	$\overline{I}$	0.1731
Deposits Interest Rate	$\overline{R^n}$	1.0101
Loans and Monetary Policy Rate	$\overline{R^L}$	1.0334
Real Deposits	$\overline{N/P}$	0.5614
Real Money Stock	$\overline{M/P}$	1.2669
Money issuance rate by the Monetary Authority (Quarterly)	x^m	-0.01
Marginal Costs	$\overline{mc}$	0.9091
Price Dispersion	$\overline{VP}$	1
Inflation Target	$\overline{\pi}$	1

Table 2: Non-stochastic Steady State.

sumption that households lose in order to be indifferent between stochastic and non-stochastic welfare. Let λ denotes this fraction of non-stochastic consumption that households give-up, in order to equal non-stochastic and stochastic welfare, such as:

$$\mathcal{U}\Big((1-\lambda)\overline{c},\overline{h}\Big) = E_t\Big\{\mathcal{U}(c_t,h_t)\Big\}. \tag{32}$$

I approximate the right-hand side of the above equation by a second-order Taylor expansion around the steady state. As the theory teaches us, there exists a cost in increasing inflation.

Figure 2 shows different welfare costs and output fluctuations (around the steady-state), while Figure 3 presents the welfare costs and inflation volatility (also inflation gap volatility against inflation steady state). Business Cycle Cost (BCC) follows a non-linear relationship.

Also, for a loss function described by Eq. 33 we observe similar results, presented by Figure 4.

$$\mathcal{L} = E_t\Bigg\{\sum_{i=0}^{\infty}\beta^i(\tilde{\pi}_{t+i}^2 + \mu\tilde{X}_{t+i}^2)\Bigg\} \tag{33}$$

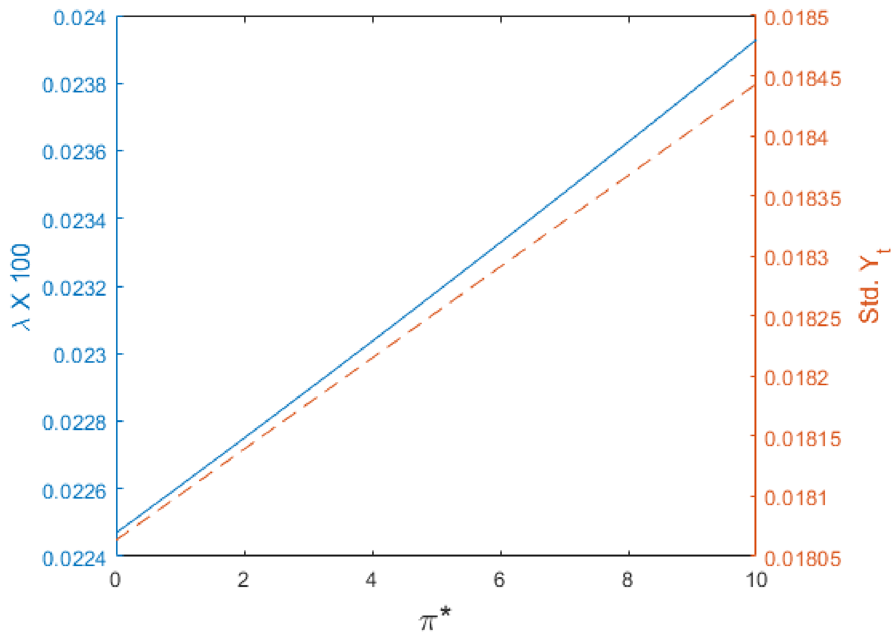

Figure 2: The Cost of Business Cycle: Welfare Cost and Output Volatility

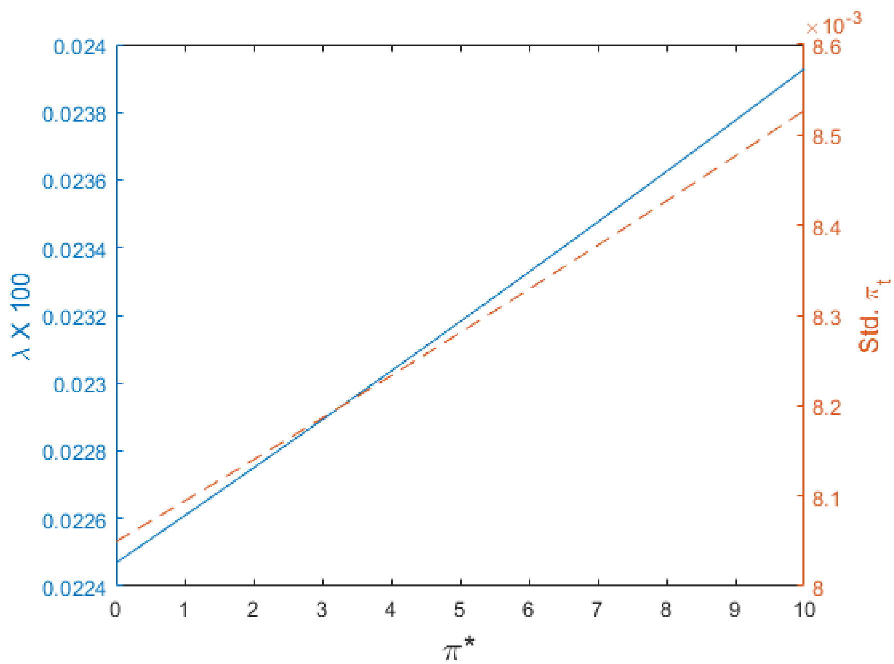

Figure 3: The Cost of Business Cycle: Welfare Cost and Inflation Volatility

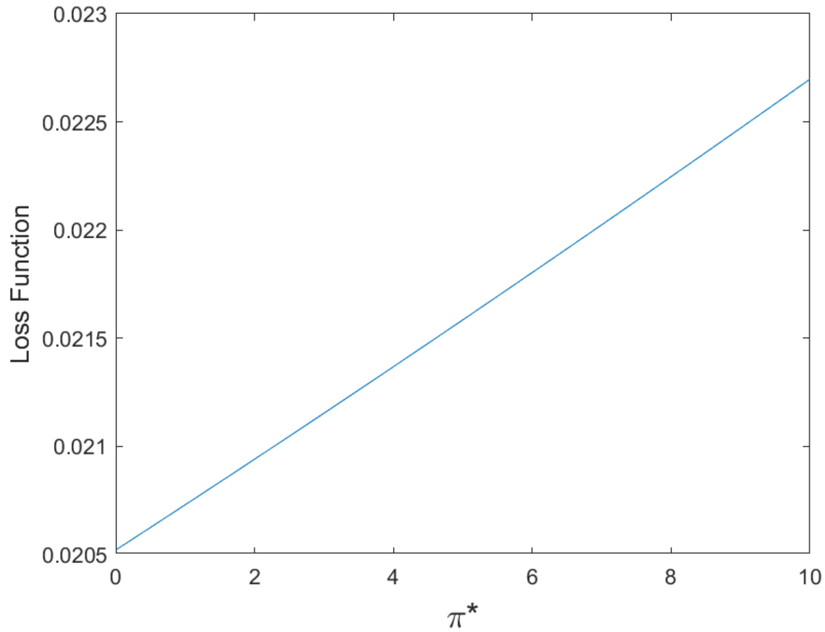

Figure 4: The Cost of Business Cycle: The Loss Function for Different Inflation Targets (π^*)

4.2 Optimal Monetary Policy given the Inflation Target

Departing from the widely accepted Taylor coefficients and in the context of simple rules, I consider the optimal monetary policy that minimizes the following loss function:

$$\begin{aligned} &\min_{\zeta_\pi,\zeta_x} E_t\Big\{\sum_{i=0}^{\infty}\beta^i(\tilde{\pi}_{t+i}^2+\mu\tilde{X}_{t+i}^2)\Big\} \\ &\text{s.t.: } A_1E_t\tilde{y}_{t+1}+A_2\tilde{y}_t+A_3\tilde{y}_{t-1}+Ce_t=0 \end{aligned} \quad (34)$$

where $\tilde{y}_t$ is the matrix of log-linearized endogenous variables, and e_t is the matrix of stochastic shocks.

Figure 5 presents the optimal monetary policy coefficients for the different yearly inflation targets. Interestingly, as the inflation target increases also increases the monetary authority's response to inflation deviations from the target. This is not surprising, since traditional economic theory and Figure 2 and 3 tell us that inflation is costly for households. Therefore, the higher the inflation target π^* (*ceteris paribus*), the higher the volatility of output [13] and the higher the volatility of inflation[14]. The response to output gap deviations is muted.

Households are risk averse and consequently do not enjoy volatility and uncertainty. Not surprisingly, the monetary authority plan to minimize 34 ends up parsimoniously increasing ζ_π, given increases in the inflation target π^*.

5 Conclusion

This paper studied optimal inflation targeting through a DSGE with microeconomics foundations, calibrated to a generic emerging market economy. The announced inflation target affects not only inflation fluctuations, but it affects the whole economy's volatility, therefore weighing on welfare. The optimal inflation target should be the welfare maximizing target, around zero percent. Also, the higher the inflation target is, the higher the monetary policy response to inflation deviation from the target, when operating optimally. Optimal monetary policy response to output gap deviations is muted, in consonance with Schmitt-Grohé and Uribe [2007].

[13] Figure 2

[14] Figure 3

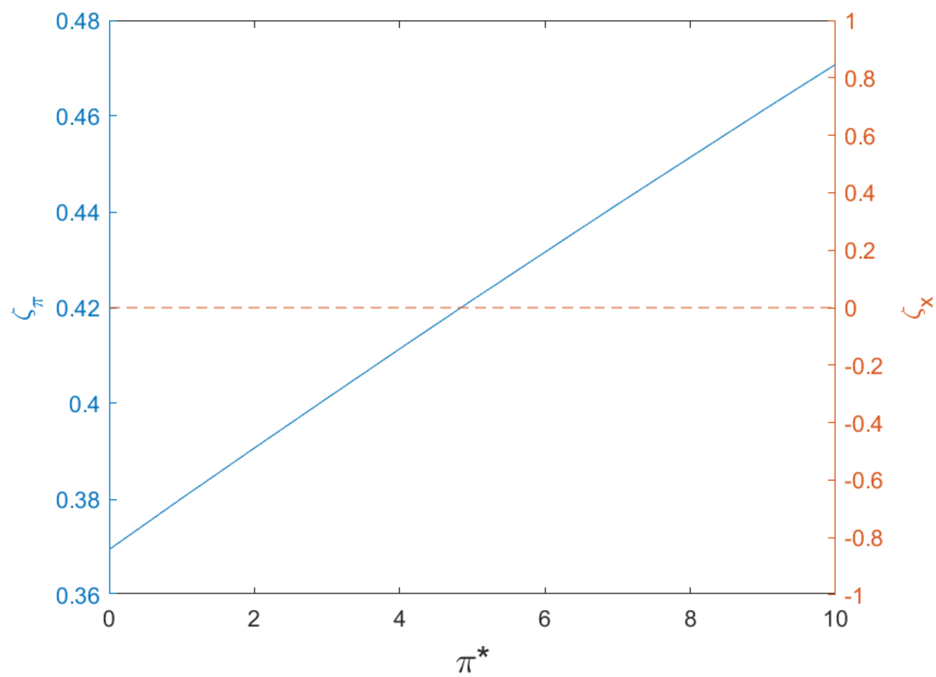

Figure 5: Optimal Monetary Policy given the Inflation Target

There are many ways this study can be amplified to better understand the optimal inflation target. In particular, adaptive learning techniques, such as presented by Slobodyan and Wouters [2012], are an attractive alternative method to study macroeconomics under limited rationality. To address the concerning issues studied at Araujo et al. [2023] regarding monetary policy under fiscal fragility one might include a fiscal sector in the model presented, with exogenous spending shocks and optimization regarding distortion tax and money issuance to fund the extra spending.

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6 Appendix

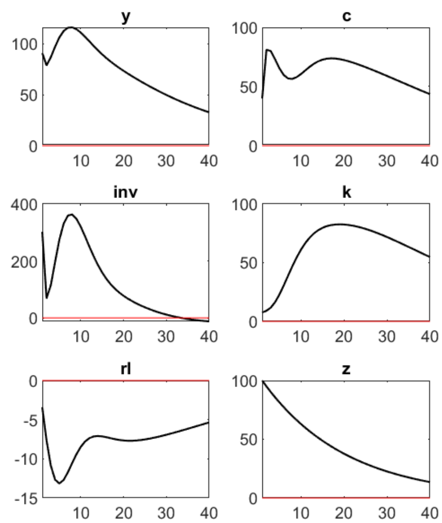

Figure 6: IRF: TFP Shock

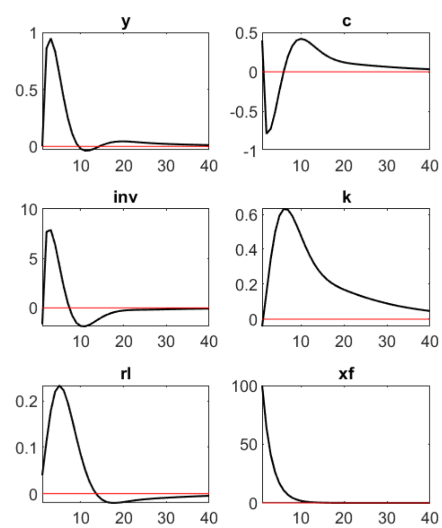

Figure 7: IRF: Money Issuance Shock

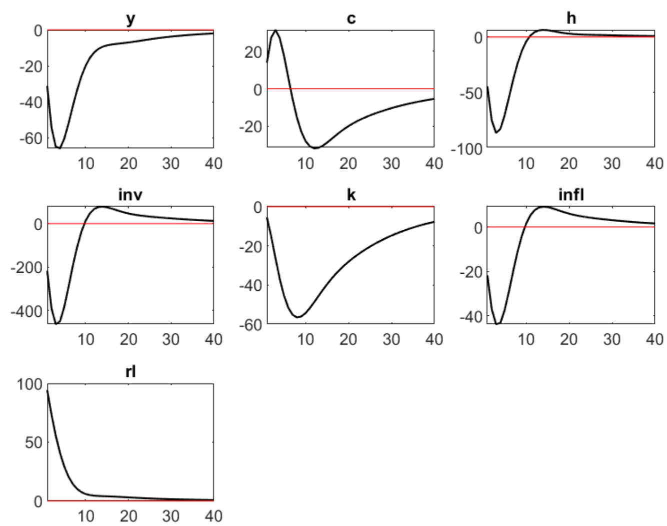

Figure 8: IRF: Monetary Policy Shock