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A Model of Global Currency Pricing
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ABSTRACT

This paper proposes a concept of a global currency and introduces a "global currency pricing" specification into a standard N-country open economy macroeconomic model. A global currency is defined as a virtual unit of account that is exclusively used for international trade invoicing and is formed as a basket of individual currencies, similar to the existing SDR. We show there is a unique optimal composition of a global currency that weights currencies according to their importance in international trade. A striking implication is that under this global currency design, the monetary policy of each country should be concerned solely with domestic shocks. No country should have more than a 50 percent weight in an optimal global currency, and a situation where a large country has the sole weighting in the global currency is likely to be worst outcome from a perspective of global welfare. We derive the conditions under which global currency pricing (GCP) dominates all other outcomes, and is an optimal choice of invoicing currency for individual firms.

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1 Introduction

The US dollar plays a significant role in international trade pricing, with a substantial share of global exports invoiced in dollars—far exceeding the United States' share of global trade (Boz et al., 2022), which has a significant impact on exchange rate transmission. As is well understood, in New Keynesian open economy (NKOE) macroeconomics, the currency of export pricing is a central issue. For example, Devereux and Engel (2003) demonstrates that optimal monetary policy exhibits different characteristics when prices are preset in producer currency (PCP) and local currency (LCP). Later, Clarida et al. (2002) and Engel (2011) show that even with dynamic pricing, the PCP and LCP pricing specification may lead to optimal monetary policy with different approaches to inflation targeting. However, there is strong evidence that supports dominant currency pricing (DCP), defined as an environment where all traded goods are priced in a single currency, with the US dollar playing this role (Gopinath et al., 2010). Given this prior research, a natural question arises: what new pricing paradigms could emerge from a international system dominated by one currency? Under what conditions might the US dollar's dominant role be challenged, at least in terms of an invoicing currency?

This paper explores the consequences of a trade pricing using a basket index comprised of multiple currencies, which we call *global currency pricing* (GCP) The global currency is a composite that resembles the IMF's Special Drawing Rights (SDRs). SDRs consist of a basket of fiat currencies, which serve as a global reserve asset. SDR's were established by the International Monetary Fund to supplement the official reserves of its member countries. Currently, SDRs comprise five major currencies: the U.S. dollar, euro, Chinese renminbi, Japanese yen, and British pound sterling.[1]

The paper takes as a given possibility that a global basket currency is created, and look at its macroeconomic implications based on New Keynesian literature (Galí, 2018). We ask how global currency differs from traditional invoicing methods like PCP, LCP, and DCP in terms of exchange rate pass-through, equilibrium allocations, and welfare, and following this, how an optimal monetary policy would operate under GCP. The basic structure of our model is quite standard. There is a large number (N) of countries of varying size. In each country there is a tradable composite good and a non-tradable good. The model is static, but we use a one-period version of Calvo pricing, where a fraction of firms have to set prices in advance and the complementary fraction can adjust their price ex-post after money or productivity shocks (Mukhin, 2022). Consumers in each country have preferences defined over all traded goods and supply labor. Given the one period nature of the analysis, we introduce monetary policy via a quantity theory relationship. The model is deliberately kept minimal so that we can conduct the analysis analytically and all our results can be obtained as simple closed form expressions.

We allow for a variety of invoicing practices over internationally traded goods, each of which has a different implication for exchange rate pass-through, international spillovers, and optimal monetary policy.

[1] The potential of a composite currency use in international trade has been suggested by Brunnermeier et al. (2021) and Carney (2019) among others.

Two versions of optimal policy analysis are discussed. In the first version, we assume a cooperative policy (or cooperative game) where a central authorities chooses national policies to minimize a global loss function similar to Engel (2011). In the second version, we assume that national policy-makers determine policy non-cooperatively (in a Nash game) to minimize a domestic country loss, following Fujiwara and Wang (2017).

A key aspect of our analysis is the tension in monetary policy between targeting domestic objectives, given that non-traded goods and domestic traded products are priced in domestic currency, and the response to international shocks which affect import consumption allocations. In our baseline model, we show that this tension may be resolved by using PCP, as it supports the well-known "divine coincidence" of NKOE models (Chari et al., 2002). However, with alternative form of currency invoicing, there remains a trade-off between the two objectives.

Our first main result is that under a cooperative Global Currency Pricing (GCP) policy, there is a unique optimal composition of the global currency basket. This basket has the property that it replicates an outcome that would be achieved if there were a completely separate trade currency independent of the monetary policies of any of the members of the basket currency. Moreover, under this composition of the global currency, each member country is free to focus purely on offsetting domestic shocks, ignoring any external influences. Strikingly, given an optimal currency composition as described by this basket, each country would follow a monetary policy identical to the policy that is optimal under traditional producer currency pricing. In other words, in the terms coined by Obstfeld and Rogoff (2002), each country would pursue a purely 'self-oriented' monetary policy.

We show that the optimal currency basket must be comprised of all N currencies, and the share of any one currency in the basket is proportional to its share of international trade, not to country size. Moreover, no country should hold more than a 50 percent share in the optimal GCP basket. A situations where the currency basket is dominated by the largest country is the worst outcome in terms of global welfare.

We go on to explore a non-cooperative or Nash scenario, where individual countries follow optimal monetary policies to minimize national losses. Again, each policy maker faces a tension between domestic and international objectives, depending on the composition of the global currency. But now we find the polar opposite result. From a global welfare point of view, it is better that larger countries have a higher weight in the global currency basket. The reason is that larger countries have a higher share of global imports and therefore have a lower cost (and higher incentive) to fashion monetary policy to respond to losses coming from the international dimension.

The model also allows a straightforward welfare comparison across alternative pricing paradigms, when policy is chosen optimally conditional on the currency of pricing. In the baseline model, we show that an equilibrium under PCP welfare dominates all other equilibria, followed by GCP, which dominates both DCP (dollar currency pricing) and LCP (local currency pricing). This is again due to the fact that optimal monetary exploits the divine coincidence. But GCP represents the welfare maximizing outcome in the case where one currency (or currency basket) is chosen as the pricing currency for all traded goods.

However, this welfare ranking is sensitive to the assumptions about monetary policy effectiveness. In Section 4, we relax the assumption that monetary policy can perfectly respond to all shocks and assume that each country experiences random monetary shocks, which may be thought of as financial or velocity shocks. We assume that these shocks cannot be directly offset by a monetary policy rule. In this case, we show that GCP may represent the welfare maximizing pricing policy, even under optimal monetary policy. In this case, the divine coincidence breaks down, and while PCP allows each monetary authority to respond optimally to country specific productivity shocks, the inability to offset velocity shocks means that the shock must be fully absorbed within each country. But with GCP, these shocks are diversified away by the composition of the global currency basket. As a result, there is a trade-off in welfare terms between the efficient elimination of real shocks (which cause currency misalignment) under PCP and the stabilizing of monetary shocks offered by the global currency.

As noted, our baseline model is kept deliberately simple so as to provide analytical insight. But a later section extends the analysis to a fully dynamic model with Calvo pricing and different assumptions about the invoicing currency as described above. Calibrating the model to 20 countries, we carry out a welfare comparison of the current dollar based invoicing system to one where the global currency basket is comprised of the SDR. We find that all countries would gain by switching from dollar invoicing to SDR invoicing, and the gains would be greatest for the US.

In section 6, we go beyond the macro analysis of global currencies and ask under what conditions individual price setters would opt to set prices in a global currency, and whether a global currency could form an equilibrium pattern of invoicing with endogenous currency invoicing decisions. Here we show that the outcome depends sensitively on the composition of shocks and the presence of complementarity in firm pricing setting. In the baseline model, with only productivity shocks, we show that if the cooperative optimal currency basket as described above were put in place, firms would tend to set prices exclusively in domestic currencies, or follow PCP. In this case PCP becomes a self-consistent equilibrium, where optimal policy follows that under PCP and firms continue to choose PCP as a pricing policy. But when we extend the model to allow for monetary shocks, we derive conditions under which GCP becomes an equilibrium outcome. We then extend the model to allow for complementarities in price setting and the presence of global input output linkages in production. We show that in both cases, it is more likely that there may exist a stable self-consistent GCP equilibrium where optimal monetary policy is based on the presence of GCP pricing and individual firms endogenously choose to set all trade prices in global currency.

1.1 Related Literature

Our paper is part of the large literature on the currency of export pricing. In this literature, three fundamental price specifications often discussed are PCP, LCP and DCP. Devereux and Engel (2003) underscore the importance of distinguishing between PCP and LCP in shaping optimal monetary policies in open economies. However, in their model, firms' pricing decisions were considered exogenous, so Devereux et al. (2004) and Bacchetta and van Wincoop (2005) incorporate the micro-level decision of firms determining invoice prices into a general equilibrium model. Subsequently, numerous studies demonstrate the impact of invoicing patterns on exchange rate pass-through (Gopinath et al., 2010; De Gregorio et al., 2024) and monetary policy transmission (Goldberg and Tille, 2009; Zhang, 2022; Cook and Patel, 2023). In addition, there are extensive studies exploring the determinants of firms' invoicing currency choices, such as exchange rate fluctuations and exchange rate regime (Goldberg and Tille, 2016), market share (Devereux et al., 2015), hedging costs (Novy, 2006; Lyonnet et al., 2022; Berthou et al., 2022), imported input reliance (Chung, 2016), input-output linkages and complementarity (Gopinath and Stein, 2021; Amiti et al., 2022). Our model closely aligns with Mukhin (2022), which uses a general equilibrium model with price stickiness to show that the dollar's dominance as an invoicing currency arises from the U.S.'s large economy advantage and the dollar's stability as an anchor currency.

More generally, there is an extensive literature on the role of the US dollar in both trade, investment, and financing. In international trade, dollar invoicing, or DCP, also referred to as the dominant currency paradigm, is extensively discussed in Gopinath and Itskhoki (2022). In the realms of financing and investment, dollar assets also have become the leading assets (Maggiori et al., 2020) due to their unique convenience yield (Jiang et al., 2021; 2024). Firms opt to issue dollar-denominated debt because it offers low interest rates (Salomao and Varela, 2022), robust liquidity (Coppola et al., 2023), stable pricing (Bocola and Lorenzoni, 2020; Drenik et al., 2022), and complements the dollar demand of other agents (Chahrour and Valchev, 2022). Investors prefer dollar assets for their strong liquidity (Engel and Wu, 2023), high safety (He et al., 2019), and countercyclical insurance properties (Hassan, 2013; Maggiori, 2017; Hassan and Zhang, 2021). The dominant role of the dollar in international trade and finance is often referred to as an 'exorbitant privilege,' which has profound implications for the global economy (Gourinchas and Rey, 2022). The U.S. may uses its market power to issue government bonds (Choi et al., 2023), often seen as safe assets and bought extensively by investors at low interest rates (Blanchard, 2019; Caramp and Singh, 2023), enabling the U.S. to sustain what some describe as Ponzi-like financial practices (Brunnermeier et al., 2024). In addition, the dollar is at the centre of the global financial cycle (Miranda-Agrippino and Rey, 2020; Obstfeld and Zhou, 2022), with U.S. interest rate hikes negatively affecting other economic systems.

Our paper is directed at a more narrow issue than the above literature. Still, to the extent that a GC offers a possible alternative to dollar invoicing, it is related to recent discussions about reducing dependency on the dollar. Recent studies have focused on the internationalization of other currencies, such as the RMB, exploring measures like establishing central bank swap lines (Bahaj and Reis, 2020), gradually liberalizing financial accounts (Clayton et al., 2024), supporting cross-border financial rescues (Horn et al., 2023), and initiating multi-country central bank digital currencies (BISpaper, 2022). Our paper explores an approach to undermine the dominance of a single currency by proposing a basket of currencies as the invoicing standard in the international trade.

The structure of the paper is as follows: Section 2 introduces the benchmark model, which incorporates exogenous global currency pricing. Section 2.2 describes the model equilibrium for given monetary policy. Section 3 investigates the nature of optimal monetary policy with 3.1 focusing on cooperative policy and the optimal design of an international currency, while 3.2 focuses on non-cooperative policy. Section 4 extends the analysis to both productivity shocks and monetary/financial shocks. Section 5 extends the model to a dynamic setting and provides a quantitative calibration. Section 6 looks at the optimal choice of invoicing currency by individual firms. Section 7 provides some conclusions.

2 A Simple Model

Our model follows the basic NKOE framework, where price stickiness enables the effectiveness of monetary policy. In order to obtain a clear closed-form solution for welfare analysis, we make many simplified assumptions in the baseline model, including log utility functions, linear labor disutility, a Cobb-Douglas aggregator, complete financial markets, no intermediate goods, and one-period Calvo price setting.

2.1 Environment

There are N countries with asymmetric sizes, where in country i there exists a continuum of n_i households and a continuum of n_i monopolistically competitive firms producing different varieties. Normalize so that $\sum_{i=1}^{N} n_i = 1$, where country 1 represents the US. This framework allows us to replicate various models in NKOE literature. For $N \to \infty$ and $n_i \to 0$, the model aligns with frameworks such as Gali and Monacelli (2005, 2008), assuming all countries are small open economies; for $N = 2$ and $n_1 + n_2 = 1$, it mirrors economies akin to Clarida et al. (2002), featuring only two major economies; and for $n_1 \to 0$ and $n_2 \to 1$, country 1 stands as a standalone small open economy like De Paoli (2009). Each country has its own fiat currency, and the exchange rate between currency i and currency j is denoted by $\mathcal{E}_{ijt}$, where an increase indicates a depreciation of currency i.

The asymmetry between countries arises not only from differences in size but also from an uneven international pricing system. We introduce a unit of account we call Global Currency, which is comprises a basket of currencies from different countries. Let $\mathcal{E}_{igt}$ denote the price of the global currency in terms of currency i. We consider the composition structure of the global currency as $(\mathcal{E}_{1gt})^{\alpha_1}(\mathcal{E}_{2gt})^{\alpha_2}\cdots(\mathcal{E}_{Ngt})^{\alpha_N} = 1$ with $\sum_{i=1}^{N} \alpha_N = 1$. Dominant currency pricing (DCP), as defined by Gopinath and Itskhoki (2022) represents a special case where the global currency is exclusively composed of the dollar, namely $\alpha_1 = 1$. It's easy to show that the exchange rate between currency i and global currency g is determined by:

$$\mathcal{E}_{igt} = (\mathcal{E}_{i1t})^{\alpha_1}(\mathcal{E}_{i2t})^{\alpha_2}\cdots(\mathcal{E}_{iNt})^{\alpha_N}.$$

Thus, α_i represents the share of currency i within the global currency basket. For instance, when $\alpha_i = 1$, we have $\mathcal{E}_{igt} = 1$, indicating that the global currency is entirely constituted by currency i.

2.1.1 Households

Preferences. In country i, the representative household obtain funds by providing labor L_{it}, investing in state contingent bonds, and receiving profits from firms, which they use for consumption bundle C_{it} and payment of lump-sum taxes T_{it}. The household's utility function is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\ln C_{it}-L_{it}\right).$$

The household's optimal labor supply satisfies $W_{it}=P_{it}C_{it}$. The stochastic discount factor between period $t-1$ and t is given by $Q_{it}=\beta(P_{it-1}C_{it-1})/(P_{it}C_{it})$.

Consumption bundles. Representative households in country i simultaneously consume both domestic and foreign goods, with a home bias v capturing the preference for domestically produced goods, which can also be interpreted as a preference for non-tradable goods. Country i's consumption bundle C_{it} consists of the non-tradable goods bundle C_{Nit} and the tradable goods bundle C_{Tit} through the Cobb-Douglas aggregator. Therefore, the consumption aggregator and the consumption price index (CPI) for region i are given as:

$$C_{it}=\frac{C_{Nit}^{v}C_{Tit}^{1-v}}{v^v(1-v)^{1-v}},\qquad P_{it}=P_{Nit}^{v}P_{Tit}^{1-v},$$

where non-tradable goods bundle C_{Nit} and price index P_{Nit} are defined by

$$C_{Nit}=\left(n_i^{-\frac{1}{\varepsilon}}\int_0^{n_i}C_{Nit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}d\omega\right)^{\frac{\varepsilon}{\varepsilon-1}},\quad P_{Nit}=\left(\frac{1}{n_i}\int_0^{n_i}P_{Nit}(\omega)^{1-\varepsilon}d\omega\right)^{\frac{1}{1-\varepsilon}}.$$

The tradable goods consumption bundle C_{Tit} is composed of the consumption baskets from country $i\in\{1,...,N\}$ by the Cobb-Douglas aggregator:

$$C_{Tit}=\Pi_{j=1}^{N}\left(\frac{C_{jit}}{n_j}\right)^{n_j},\quad P_{Ti}=\Pi_{j=1}^{N}(P_{jit})^{n_j},$$

where P_{jit} is the price of tradable goods from country j to country i denominated in currency i. Notice that the tradable goods bundle C_{jit} and price index P_{jit} are adjusted by n_j, since there is only a continuum of n_j monopolistically competitive firms in country j. Larger countries produce more goods and thus hold a larger share in tradable goods.

The tradable goods bundle C_{jit} and price index P_{jit} are defined by

$$C_{jit}=\left(n_j^{-\frac{1}{\varepsilon}}\int_0^{n_j}C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}d\omega\right)^{\frac{\varepsilon}{\varepsilon-1}},\quad P_{jit}=\left(\frac{1}{n_j}\int_0^{n_j}P_{jit}(\omega)^{1-\varepsilon}d\omega\right)^{\frac{1}{1-\varepsilon}},$$

where $\varepsilon>1$ represents the elasticity of substitution between varieties.

Complete Markets. We focus on the simplest complete markets setup, where there are state-contingent bonds traded in each period, thus satisfying the risk-sharing condition:

$$\mathcal{E}_{ijt}=\eta_{ij}\frac{P_{it}C_{it}}{P_{jt}C_{jt}}.\tag{2.1}$$

Since log utility is used in our model, we have $\eta_{ij}=1$ in equilibrium. [2]

[2] The proof can be found in the appendix of Devereux and Engel (2003).

2.1.2 Firms

Following Mukhin (2022), our model employs a one-period version of Calvo (1983) price setting to characterize price stickiness. Specifically, in each period t, a fraction θ of firms can only set goods prices at the beginning of each period t, while the remaining fraction $1-\theta$ can adjust prices after the realization of all shocks. This assumption bridges between one-period in advance (e.g., Obstfeld and Rogoff, 1995, 1998) and standard Calvo price setting (e.g., Clarida et al., 2000). The advantage is that θ quantifies the degree of price stickiness, but because prices are sticky for only one period, the measure of preset prices is independent of its history.[3]

In the baseline model, we abstract from intermediate goods, and assume production functions are linear in labor, so that marginal cost for non-tradable and tradable firms in country j is simply $MC_{jt} = W_{jt}/Z_{jt}$. The profit function of θ proportion of the sticky-price firm ω producing non-tradable goods in country j is:

$$\max_{\bar{P}_{Njt}(\omega)} E_{t-1}\left\{Q_{jt}\left[\left(\bar{P}_{Njt}(\omega)-(1-\tau_{jt})MC_{jt}\right)\left(n_j Y_{Njt}(\omega)\right)\right]\right\},$$

A tradable firm ω in country j sets two different prices for the domestic and foreign markets, $P_{jjt}(\omega)$ and $P^G_{j-jt}(\omega)$, denominated in currency j and the global currency respectively. The optimization problem for the sticky GCP firm is therefore:

$$\max_{\bar{P}_{jjt}(\omega)} E_{t-1}\left\{Q_{jt}\left[\left(\bar{P}_{jjt}(\omega)-(1-\tau_{jt})MC_{jt}\right)\left(n_j Y_{jjt}(\omega)\right)\right]\right\},$$

$$\max_{\bar{P}^G_{j-jt}(\omega)} E_{t-1}\left\{Q_{jt}\left[\left(\mathcal{E}_{jgt}\bar{P}^G_{j-jt}(\omega)-(1-\tau_{jt})MC_{jt}\right)\left(\sum_{i\neq j} n_i Y_{jit}(\omega)\right)\right]\right\},$$

We multiply $Y_{jit}(\omega)$ by n_i since there is a continuum of n_i households in country i, meaning that larger countries have a higher demand for variety ω.

To eliminate the monopolistic distortion in equilibrium, we assume that $\tau_{jt} = 1/\varepsilon$ is an exogenous subsidy provided by the government. Hence, the price set by the firm at the beginning of period t is:

$$\bar{P}_{Njt} = \frac{E_{t-1}\left(Q_{jt}MC^N_{jt}(P_{Njt})^{\varepsilon}Y_{Njt}\right)}{E_{t-1}\left(Q_{jt}(P_{Njt})^{\varepsilon}Y_{Njt}\right)},$$

$$\bar{P}_{jjt} = \frac{E_{t-1}\left(Q_{jt}MC_{jt}(P_{jjt})^{\varepsilon}Y_{jjt}\right)}{E_{t-1}\left(Q_{jt}(P_{jjt})^{\varepsilon}Y_{jjt}\right)},$$

$$\bar{P}^G_{j-jt} = \frac{E_{t-1}\left(Q_{jt}MC_{jt}(P^G_{j-jt})^{\varepsilon}\left(\sum_{i\neq j} n_i Y_{jit}\right)\right)}{E_{t-1}\left(Q_{jt}\mathcal{E}_{jgt}(P^G_{j-jt})^{\varepsilon}\left(\sum_{i\neq j} n_i Y_{jit}\right)\right)}.$$

Additionally, there is a proportion of $1-\theta$ firms that set prices flexibly as follows:

$$\tilde{P}_{Njt} = MC_{jt}, \quad \tilde{P}_{jjt} = MC_{jt}, \quad \tilde{P}^G_{j-jt} = MC_{jt}\mathcal{E}_{gjt}.$$

In equilibrium, the price for non-tradables is given by $(P_{Njt})^{1-\varepsilon} = \theta(\bar{P}_{Njt})^{1-\varepsilon} + (1-\theta)(\tilde{P}_{Njt})^{1-\varepsilon}$, and similarly for P_{jjt} and P^G_{j-jt}.

[3] The baseline model, which uses one-period Calvo pricing, can be simplified into a one-period static model. Since we will extend the model to standard Calvo pricing in an extension, we maintain a dynamic framework in the baseline model.

2.1.3 Monetary Policy

Following some money-in-utility models or cash-in-advance models, we assume the money demand in country i satisfies $M_{it} = P_{it}C_{it}$. In the discussion below, the money supply M_{it} will be treated as a policy instrument, with a committed central bank selecting it in response to various exogenous productivity shocks.

2.1.4 Equilibrium Conditions

Definition of Equilibrium. Given the stochastic process of the external productivity shocks Z_{it} and monetary policies M_{it}, the competitive equilibrium satisfies (a) households optimally choose consumption, state-contingent assets, and labor supply; (b) firms maximize profits; (c) goods and labor markets clear as follows:

$$n_j L_{jt} = \frac{1}{Z_{jt}} n_j C_{Njt}\Delta_{Njt} + \frac{1}{Z_{jt}} n_j C_{jjt}\Delta_{jjt} + \frac{1}{Z_{jt}} \sum_{i \neq j} n_i C_{jit}\Delta^G_{j-jt}, \tag{2.2}$$

where $\Delta_{Njt} = \frac{1}{n_j}\int_0^{n_j} (\frac{P_{Njt}(\omega)}{P_{Njt}})^{-\varepsilon} d\omega$, $\Delta_{jjt} = \frac{1}{n_j}\int_0^{n_j} (\frac{P_{jjt}(\omega)}{P_{jjt}})^{-\varepsilon} d\omega$, and $\Delta^G_{j-jt} = \frac{1}{n_j}\int_0^{n_j} (\frac{P^G_{j-jt}(\omega)}{P^G_{j-jt}})^{-\varepsilon} d\omega$ is the price dispersion term. The market clearing condition indicates that labor in country j is used for the production of domestic non-tradable goods C_{Njt} and tradable goods consumed domestically C_{jjt} and exported abroad C_{jit}. The appendix outlines the equilibrium system and first-order logarithmic linearization for PCP, LCP, DCP, and GCP.

2.2 Equilibrium

In our paper, we use lowercase y_{ijt} denote the log deviation from the steady state $\ln Y_{ijt} - \ln Y_{ij,ss}$.[4] Log productivity shocks z_{it} are assumed to follow a normal distribution $N(0, \sigma^2_{iz})$, and the shocks between countries are independent. For the money supply shock m_{it}, if treated as exogenous, its mean is also assumed to follow normal distribution. When the committed central bank actively implements monetary policy, m_{it} becomes a linear function of other shocks, ensuring that $E_{t-1}m_{it} = 0$ also. We further define the size-weighted global shock as $x_t = \sum_{i=1}^N n_i x_{it}$ for shocks $x_{it} \in \{z^N_{it}, z_{it}, m_{it}, mc_{it}\}$.

2.2.1 Exchange Rate Pass-Through

Exchange Rate. Under the assumption of complete markets, the bilateral exchange rate e_{ijt} can be considered exogenous shocks due to the risk sharing condition equation (2.1.1):

$$e_{ijt} = m_{it} - m_{jt}.$$

This implies that the exchange rate between two countries depends only on their relative money supply.

We further define a virtual money supply $m_{gt} = \sum_{i=1}^N \alpha_i m_{it}$, which is a weighted average of each country's money supply based on their share in the global currency basket. It is as if there exists a virtual

[4] With the one-period Calvo pricing setting, we can also define the log deviation as $y_{ijt} = \ln Y_{ijt} - E_{t-1}\ln Y_{ijt}$, avoiding the need to assume expectations satisfying $E_{t-1}x_t = 0$.

country g in the economy, whose money supply depends on the monetary policy of the countries in the global currency basket, and the exchange rate between currency i and the global currency g is:

$$e_{igt} = \sum_{j=1}^{N} (\alpha_j e_{ijt}) = m_{it} - m_{gt}.$$

The depreciation of currency i relative to the global currency g depends on whether country i's money supply exceeds that of the virtual country g. When $\alpha_1 = 1$, meaning the global currency is entirely composed of the dollar, this virtual country g can be considered as the United States/country 1.

Exchange Rate Pass-Through. Under the four pricing paradigms, the currency i price of tradable goods exported from country j to country i, p_{jit}, is given by:

$$\begin{aligned} PCP &: p_{jit} = (1-\theta)mc_{jt} + e_{ijt}; \\ LCP &: p_{jit} = (1-\theta)mc_{jt} + (1-\theta)e_{ijt}; \\ DCP &: p_{jit} = (1-\theta)mc_{jt} + (1-\theta)e_{ijt} + \theta e_{i1t}; \\ GCP &: p_{jit} = (1-\theta)mc_{jt} + (1-\theta)e_{ijt} + \theta \sum_{k=1}^{N} \alpha_k e_{ikt}. \end{aligned}$$

The above equations show how the exchange rate is transmitted to consumer prices. It is only when prices are sticky, i.e., $\theta \neq 0$, that the different pricing strategies have different effects in the economy. When $\theta = 1$, the pass-through of exchange rates to prices varies significantly: Under PCP, there is complete pass-through of exchange rates from other countries; under LCP, there is no pass-through; under DCP, there is complete pass-through of the dollar exchange rate; and under GCP, each country's currency k passes through to prices p_{jit} in proportion to α_k.

2.2.2 Allocation Deviation

Efficient allocation. Since our model uses taxes to eliminate the distortion from monopolistic competition the flexible price equilibrium is efficient (Galí, 2008; Woodford and Walsh, 2005). We use a tilde, $\tilde{y}_{ijt}$, to denote the log deviation of the efficient allocation from the steady state, $\ln \tilde{Y}_{ijt} - \ln Y_{ij,ss}$, where $\tilde{Y}_{ijt}$ is the efficient allocation. Thus, we have:

$$\tilde{c}_{it} = \tilde{y}_{it} = v z_{it} + (1-v) z_t, \quad \tilde{c}_{Nit} = z_{it}, \quad \tilde{c}_{Tit} = z_t, \quad \tilde{c}_{jit} = z_{jt}.$$

For the first-best allocation, consumption is fully determined by productivity shocks.

Consumption deviation. When nominal rigidity distorts the economy, monetary policy can influence the equilibrium allocation. Following Itskhoki and Mukhin (2023), we describe the equilibrium by its log deviation from the first-best allocation. The non-tradable consumption c_{Nit} and domestic tradable consumption c_{iit} in country i depend on m_{it}, as both goods are priced in currency i:

$$c_{Nit} - \tilde{c}_{Nit} = c_{iit} - \tilde{c}_{iit} = \theta(m_{it} - z_{it}); \tag{2.3}$$

However, international trade is more complex, depending on the pricing paradigm, as households' consumption depends on the money supply of the currency in which those goods are priced:

$$\begin{aligned} PCP &: c_{jit} - \tilde{c}_{jit} = \theta(m_{jt} - z_{jt}); \\ LCP &: c_{jit} - \tilde{c}_{jit} = \theta(m_{it} - z_{jt}); \\ DCP &: c_{jit} - \tilde{c}_{jit} = \theta(m_{1t} - z_{jt}); \\ GCP &: c_{jit} - \tilde{c}_{jit} = \theta(m_{gt} - z_{jt}). \end{aligned} \tag{2.4}$$

For PCP, LCP, DCP, and GCP, the deviation $c_{jit} - \tilde{c}_{jit}$ depends on its pricing currency, specifically m_{jt}, m_{it}, m_{1t}, and m_{gt}. As is later discussed, this currency misalignment leads to inconsistencies in monetary policy objectives when addressing external shocks.

3 Optimal Monetary Policy

In this section, two versions of optimal monetary policy under different pricing regimes are discussed. Section 3.1 examines the case where countries are willing to cooperate to maximize global welfare, while Section 3.2 focuses on a Nash equilibrium where monetary policies are designed to maximize each country's own utility.

3.1 Cooperative Monetary Policy

We examine the differences in optimal monetary policy under the four pricing paradigms in a cooperative setting. The Appendix shows that the global loss function may be expressed as follows:

$$\delta\, \mathbb{E} \left(v \sum_{i=1}^{N} n_i (c_{Nit} - \tilde{c}_{Nit})^2 + (1-v) \sum_{i=1}^{N} n_i^2 (c_{iit} - \tilde{c}_{iit})^2 + (1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (c_{jit} - \tilde{c}_{jit})^2 \right), \tag{3.1}$$

where $\delta = (\varepsilon(1-\theta)+\theta)/(2\theta)$ is a constant related to the degree of price stickiness θ. The separation between the second and third terms inside the parentheses emphasizes that domestically produced and imported traded goods may be priced in different currencies. Substituting the consumption deviations, i.e. equation (2.4), under PCP, LCP, DCP, and GCP into equation (3.1), we obtain the global loss function. We define the common component $\mathcal{L}^c$ to the loss function which prevails across all specifications as:

$$\mathcal{L}^c = \underbrace{v \sum_{i=1}^{N} n_i (m_{it} - z_{it})^2}_{\text{related to } (c_{Nit} - \tilde{c}_{Nit})^2} + \underbrace{(1-v) \sum_{i=1}^{N} n_i^2 (m_{it} - z_{it})^2}_{\text{related to } (c_{iit} - \tilde{c}_{iit})^2}$$

The common component to all welfare losses is associated with domestic shocks that affect the non-traded consumption sector and domestic traded goods consumption equally. For losses associated with consumption of foreign tradable goods however, the global loss function differs depending on the pricing specification. We illustrate the global loss function for the four different pricing specifications as follows:

$$PCP : \kappa\, \mathbb{E} \left(\mathcal{L}^c + (1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{jt} - z_{jt})^2 \right), \tag{3.2}$$

$$LCP : \kappa\, \mathbb{E} \left(\mathcal{L}^c + (1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{it} - z_{jt})^2 \right), \tag{3.3}$$

$$DCP : \kappa\, \mathbb{E} \left(\mathcal{L}^c + (1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{1t} - z_{jt})^2 \right), \tag{3.4}$$

$$GCP : \kappa\, \mathbb{E} \left(\mathcal{L}^c + (1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{gt} - z_{jt})^2 \right), \tag{3.5}$$

where $\kappa = \theta(\varepsilon(1-\theta)+\theta)/2$ is a constant. We consider committed monetary policy, meaning that at the beginning of the period, the policy maker commits to making m_{it} a linear function of all shocks to minimize global losses. The occurrence of idiosyncratic shocks means that each country experiences different domestic shocks, with $z_{it} \neq z_{jt}$. If the global economy faced identical shocks, i.e., $z_t = z_{it} = z_{jt}$ for $\forall i, j$, then all four pricing paradigms could achieve an efficient allocation, provided that $m_{it} = z_t$ for $\forall i$. However, due to the existence of various shocks, the outcome deviates from the efficient allocation.

Equations (3.2) to (3.5) show that the key distortion in terms of welfare losses is the currency misalignment caused by LCP, DCP, and GCP pricing paradigms. Then it follows that the optimal monetary policy will depend on the various degrees of misalignment. We present the optimal policies in turn.

Producer currency pricing. The optimal monetary policy under PCP can be treated as the efficient benchmark, which is the open economy version of the 'divine coincidence' (Blanchard and Galí, 2007). We slightly rearrange the global welfare equation in equation (3.2) as:

$$PCP : \kappa\, \mathbb{E} \left(\underbrace{v \sum_{i=1}^{N} n_i (m_{it} - z_{it})^2 + (1-v) \sum_{i=1}^{N} n_i^2 (m_{it} - z_{it})^2}_{\mathcal{L}^c, \quad \text{related to } (c_{Nit} - \tilde{c}_{Nit})^2 \text{ and } (c_{iit} - \tilde{c}_{iit})^2} + \underbrace{(1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{it} - z_{it})^2}_{\text{related to } (c_{ijt} - \tilde{c}_{ijt})^2} \right). \tag{3.6}$$

Under PCP, country i's monetary policy m_{it} influences three parts: ① country i's non-tradable goods $(c_{Nit} - \tilde{c}_{Nit})^2$, ② country i's domestic tradable goods $(c_{iit} - \tilde{c}_{iit})^2$,and ③ country i's exports $(c_{ijt} - \tilde{c}_{ijt})^2$. The first two terms are part of the common component $\mathcal{L}^c$, while the third is specific to PCP pricing. All three require the monetary policy m_{it} to target the productivity shock z_{it}. Therefore, the optimal monetary policy $m_{it} = z_{it}$ achieves the efficient allocation in both the non-tradable allocation and tradable allocation. The PCP optimal monetary policy can be decomposed as a weighted sum of these three components:

$$m_{it}^{opt,cP} = \frac{v n_i}{\Delta_i^{cP}} z_{it} + \frac{(1-v) n_i^2}{\Delta_i^{cP}} z_{it} + \frac{(1-v) \sum_{j \neq i} n_i n_j}{\Delta_i^{cP}} z_{it} = z_{it},$$

where the superscript "cP" represents "cooperative outcome" and "PCP", and $\Delta_i^{cP} = v n_i + (1-v) n_i^2 + (1-v) \sum_{j \neq i} n_i n_j$ is the sum of the weights, representing the coefficients of m_{it} in the global loss function equation (3.6). Although this decomposition may appear somewhat cumbersome, it illustrates the trade-offs faced by policy makers under PCP, who must assign different importances to non-tradable goods, domestic tradable goods, and foreign tradable goods, with weights of v, $(1-v)n_i$, and $(1-v)(1-n_i)$, respectively.

Local currency pricing. Under LCP, there is currency misalignment, and the global loss function is repeated below as follows:

$$LCP : \kappa \, \mathbb{E} \left(\underbrace{v \sum_{i=1}^{N} n_i (m_{it} - z_{it})^2 + (1-v) \sum_{i=1}^{N} n_i^2 (m_{it} - z_{it})^2}_{\mathcal{L}^c, \quad \text{related to } (c_{Nit} - \tilde{c}_{Nit})^2 \text{ and } (c_{iit} - \tilde{c}_{iit})^2} + \underbrace{(1-v) \sum_{i=1}^{N} \sum_{j \neq i} n_i n_j (m_{it} - z_{jt})^2}_{\text{related to } (c_{jit} - \tilde{c}_{jit})^2} \right). \tag{3.7}$$

Under LCP, country i's monetary policy again m_{it} influences three parts: the two parts of the common component $\mathcal{L}^c$, and the third part, representing country i's imports $(c_{jit} - \tilde{c}_{jit})^2$. The first two terms are the same as under PCP, requiring m_{it} to target the domestic productivity shock z_{it}. However, the third part of the loss function requires m_{it} to target all foreign productivity shock z_{jt} for $\forall j$, since the consumption of country i's households' import consumption of good j is influenced by country i's monetary policy. Thus, country i's optimal monetary policy must balance the trade-off between domestic and foreign productivity shocks:

$$\begin{aligned} m_{it}^{opt,cL} &= \frac{v n_i}{\Delta_i^{cL}} z_{it} + \frac{(1-v) n_i^2}{\Delta_i^{cL}} z_{it} + \sum_{j \neq i} \left(\frac{(1-v) n_i n_j}{\Delta_i^{cL}} z_{jt} \right) \\ &= v z_{it} + (1-v) n_i z_{it} + (1-v) (\sum_{j \neq i} n_j z_{jt}), \end{aligned} \tag{3.8}$$

where $\Delta_i^{cL} = v n_i + (1-v) n_i^2 + (1-v) \sum_{j \neq i} n_i n_j$ is the sum of the weights. A policy maker in country i aiming to maximize global welfare must assign weights of v, $(1-v)n_i$, and $(1-v)n_j$ to non-tradable goods, domestic tradable goods, and foreign tradable goods from country j, respectively. And the monetary policy of country i will allocate a larger share to bigger countries.

Dollar currency pricing. In DCP, country 1 (the U.S.) and other countries exhibit asymmetry in their monetary policies. By slightly rearranging the equation (3.4), we have

$$DCP : \kappa \, \mathbb{E} \left(\underbrace{v \sum_{i=1}^{N} n_i (m_{it} - z_{it})^2 + (1-v) \sum_{i=1}^{N} n_i^2 (m_{it} - z_{it})^2}_{\mathcal{L}^c, \quad \text{related to } (c_{Nit} - \tilde{c}_{Nit})^2 \text{ and } (c_{iit} - \tilde{c}_{iit})^2} + \underbrace{(1-v) \sum_{i=1}^{N} n_i (1 - n_i) (m_{1t} - z_{it})^2}_{\text{related to } (c_{ijt} - \tilde{c}_{ijt})^2} \right). \tag{3.9}$$

We first examine the monetary policies of country i, $i \neq 1$. Clearly, country i's monetary policy m_{it} only influences $\mathcal{L}^c$, namely country i's non-tradable goods and domestic tradable goods, and both of which require the monetary policy m_{it} to target z_{it}. The third term is completely unrelated to the country i's monetary policy m_{it}, since country i's monetary policy cannot influence international trade. Thus, the monetary policy of non-U.S. countries only needs to ensure that non-tradable goods and domestic tradable goods reach their efficient levels:

$$m_{it}^{opt,cD} = \frac{v n_i}{\Delta_i^{cD}} z_{it} + \frac{(1-v) n_i^2}{\Delta_i^{cD}} z_{it} = z_{it} \quad \text{for } i \neq 1, \tag{3.10}$$

where $\Delta_i^{cD} = v n_i + (1-v) n_i^2$ is the sum of the weights. Hence for the non-DCP countries, the optimal policy is simply to target domestic productivity shocks, as in the PCP case.

The monetary policy of country 1 can influence four parts: ① country 1's non-tradable goods $(c_{N1t} - \tilde{c}_{N1t})^2$, ② country 1's domestic tradable goods $(c_{11t} - \tilde{c}_{11t})^2$, ③ country 1's exports $(c_{1jt} - \tilde{c}_{1jt})^2$, and ④ country i's $(i \neq 1)$ exports $(c_{ijt} - \tilde{c}_{ijt})^2$. The first three parts require m_{1t} to target z_{1t}, while the fourth part requires m_{1t} to target the productivity shocks of other countries, z_{it}. Thus we have

$$m_{1t}^{opt,cD} = \frac{vn_1}{\Delta_1^{cD}} z_{1t} + \frac{(1-v)n_1^2}{\Delta_1^{cD}} z_{1t} + \frac{(1-v)n_1(1-n_1)}{\Delta_1^{cD}} z_{1t} + (1-v) \sum_{j \neq 1} \left(\frac{n_j(1-n_j)}{\Delta_1^{cD}} z_{jt} \right), \tag{3.11}$$

where $\Delta_1^{cD} = vn_1 + (1-v)n_1^2 + (1-v)n_1(1-n_1) + (1-v)\sum_{j \neq 1} n_j(1-n_j)$. In a cooperative outcome, U.S. monetary policy takes on the role of managing global exports, leading to multiple objectives for U.S. monetary policy. In contrast, the monetary policies of other countries only need to target their own domestic productivity shocks.

Bitcoin currency pricing. Before discussing GCP, it is useful to define a hypothetical arrangement we refer to as Bitcoin currency pricing. Imagine that in addition to the N national fiat currencies, there exists a Bitcoin-like currency with a money supply m_{bt} that is independent of all other currencies. Assume that all imports and exports for all N countries are priced in Bitcoin, while domestic consumption does not use Bitcoin for pricing. We emphasize that this currency is not meant to be real, but is defined simply to help exposit the optimal policy under GCP. The global welfare function under BCP is:

$$BCP : \kappa\, \mathbb{E} \left(\underbrace{v \sum_{i=1}^{N} n_i (m_{it} - z_{it})^2 + (1-v) \sum_{i=1}^{N} n_i^2 (m_{it} - z_{it})^2}_{\mathcal{L}^c, \quad \text{related to } (c_{Nit} - \tilde{c}_{Nit})^2 \text{ and } (c_{iit} - \tilde{c}_{iit})^2} + \underbrace{(1-v) \sum_{i=1}^{N} n_i (1-n_i)(m_{bt} - z_{it})^2}_{\text{related to } (c_{ijt} - \tilde{c}_{ijt})^2} \right). \tag{3.12}$$

Under BCP, it is obvious that country i's monetary policy satisfies

$$m_{it}^{opt,cB} = \frac{vn_i}{\Delta_i^{cB}} z_{it} + \frac{(1-v)n_i^2}{\Delta_i^{cB}} z_{it} = z_{it} \quad \text{for } \forall i$$

where $\Delta_i^{cB} = vn_i + (1-v)n_i^2$. The monetary policy m_{it} only affects the common component of the global loss function $\mathcal{L}^c$. Hence the consumption of non-tradable goods and domestic tradable goods in each country reaches the efficient level.

By contrast, the Bitcoin issuer must instead weight the productivity shocks across all countries to balance the asymmetric productivity shocks as follows:

$$m_{bt}^{opt,cB} = \sum_{i=1}^{N} \left(\frac{n_i(1-n_i)}{\sum_{j=1}^{N} n_j(1-n_j)} z_{it} \right). \tag{3.13}$$

The term $n_i(1-n_i)$ is a measure of country i's international trade importance, serving as an index that captures both country i's export share n_i and import share $1-n_i$. Both extremely large and extremely small countries are likely to be overlooked, as large countries are less reliant on import consumption, while small countries' export production is too negligible to have a significant impact, resulting in a relatively small trade importance index $n_i(1-n_i)$.

We introduce BCP because it represents a fictional pricing paradigm whose welfare lies between that of PCP and GCP. This would be the optimal allocation achievable when one separate currency is used as the invoicing currency, and gives national monetary policies independence to target domestic shocks.

Welfare under BCP is strictly inferior to that under PCP because PCP achieves the optimal allocation in all three areas: non-tradable goods, domestic tradable goods, and foreign tradable goods, whereas BCP can only achieve the full optimum in the first two areas. BCP is no worse than GCP, as it can be viewed as the optimal allocation achievable when only one currency is used as the international trade invoicing currency. In other words, under DCP and GCP, the economy is constrained by having only N monetary policy instruments to address shocks, whereas BCP provides $N+1$ instruments. Consequently, BCP represents the upper bound of GCP in terms of policy effectiveness.

Global currency pricing. Now we focus on the optimal monetary policy under GCP. The global loss function under GCP is given by:

$$GCP : \kappa\, \mathbb{E}\left(\underbrace{v\sum_{i=1}^{N} n_i(m_{it}-z_{it})^2 + (1-v)\sum_{i=1}^{N} n_i^2(m_{it}-z_{it})^2}_{\mathcal{L}^c,\quad \text{related to } (c_{Nit}-\tilde{c}_{Nit})^2 \text{ and } (c_{iit}-\tilde{c}_{iit})^2} + \underbrace{(1-v)\sum_{i=1}^{N}\left(n_i(1-n_i)\left(\sum_{j=1}^{N}\alpha_j m_{jt} - z_{it}\right)^2\right)}_{\text{related to } (c_{ijt}-\tilde{c}_{ijt})^2} \right). \tag{3.14}$$

The final term in the loss function captures the fact that under GCP, to the extent that it forms part of the global currency basket, each country's monetary policy influences both its exports, captured by the n_i coefficient, and its imports, captured by the $1-n_i$ coefficient. In this case, optimal monetary policy becomes more complex because the marginal effect of m_{it} is no longer linear.

However, under GCP the optimal monetary policy can be characterized indirectly by the following expressions:

$$\begin{aligned} m_{it}^{opt,cG} &= z_{it} + \frac{a(1-v)\alpha_i}{vn_i + (1-v)n_i^2}(m_{bt}^{opt,cB} - m_{gt}^{opt,cG}), \\ m_{gt}^{opt,cG} &= \frac{\sum_{i=1}^{N}\alpha_i z_i + ab(1-v)m_{bt}^{opt,cB}}{1+ab(1-v)}, \end{aligned} \tag{3.15}$$

where $a = \sum_{i=1}^{N} n_i(1-n_i)$ and $b = \sum_{i=1}^{N}(\alpha_i^2/(vn_i + (1-v)n_i^2))$. Country i's optimal monetary policy essentially involves balancing two objectives: one is to maintain the efficiency of its own non-tradable and domestic tradable consumption, which requires m_{it} to reach z_{it}; the other is to ensure that global trade reaches the level of BCP, necessitating m_{gt} to converge to $m_{bt}^{opt,cB}$ as shown in equation (3.13). To achieve the latter objective, currencies within the global currency basket must ignore to some degree their domestic consumption efficiency $(c_{Nit} - \tilde{c}_{Nit})^2$ and $(c_{iit} - \tilde{c}_{iit})^2$ in pursuit of this global trade alignment.

Despite the complex form of equation (3.15), the variable $m_{gt}^{opt,cG}$ is the same for all N countries. This highlights two important points: first, currencies with a larger share ($\alpha_i \uparrow$) in the global currency basket deviate more from z_{it} compared to those with smaller shares. If currency i is not included in the global currency basket, i.e., $\alpha_i = 0$, then $m_{it}^{opt,cG} = z_{it}$, since currency i only needs to address deviations in non-tradable goods and domestic tradable goods. Secondly, larger countries ($n_i \uparrow$) deviate less from z_{it} compared to smaller countries from equation (3.15). This is because, for large countries, domestic consumption constitutes a significant portion of global welfare.

We introduce the concept of **positive externality of global currency under the cooperative game**: all countries share the same objective of making m_{gt} target $m_{bt}^{opt,cB}$ to achieve the optimal allocation for international trade $(c_{ijt} - \tilde{c}_{ijt})^2$. To obtain this goal, currencies in the global currency basket must sacrifice domestic consumption $(c_{Nit} - \tilde{c}_{Nit})^2$ and $(c_{iit} - \tilde{c}_{iit})^2$ to better coordinate trade allocations. Currencies not in the global currency basket or those with smaller shares can allow their monetary policies to focus more on achieving efficient $\mathcal{L}^c$.

Optimal global currency design. To eliminate the positive externality in the cooperative equilibrium, we present the optimal design of the global currency as follows:

Proposition 1 *The optimal global currency design under a cooperative game involves the participation of every country in the global currency, with country i's share α_i determined such that it satisfies:*

$$\alpha_i^* = \frac{n_i(1 - n_i)}{\sum_{j=1}^{N} n_j(1 - n_j)}.$$

Under the optimal global currency design, country i's optimal monetary policy $m_{it}^{opt,cG}$ and global currency supply $m_{gt}^{opt,cG}$ is equal to:

$$m_{it}^{opt,cG} = z_{it},$$
$$m_{gt}^{opt,cG} = \sum_{i=1}^{N} \left(\frac{n_i(1 - n_i)}{\sum_{j=1}^{N} n_j(1 - n_j)} z_{it} \right),$$

which implies that the optimal global currency design replicates the allocation in Bitcoin currency pricing as equation (3.13).

This is an interesting conclusion. Under cooperative policy, the optimal global currency design $(\alpha_1^*, ..., \alpha_N^*)$ has two key features: first, all countries are included in the global currency basket; second, the weight of country i in the global currency is determined by its trade importance index, weighted by $n_i(1-n_i)$. Including all N countries in the global currency basket provides more monetary policy tools, facilitating cooperation to drive m_{gt} closer to $m_{bt}^{opt,cB}$, and thereby reducing the need to sacrifice domestic consumption. Furthermore, it is optimal to weight each country by its trade importance index $n_i(1 - n_i)$ to exploit the positive externalities existing in the global currency pricing, giving countries with a greater trade importance index the incentive to steer m_{gt} towards the optimal level. We note also that this weighting scheme tends to underweight larger countries and overweight smaller countries, relative to their share in world GDP.

Proposition 1 also demonstrates that under this global currency design, GCP replicates the allocation of BCP. Countries' domestic consumption, $\mathcal{L}^c$, reaches the efficient level since $m_{it} = z_{it}$, while the import consumption $(c_{ijt} - \tilde{c}_{ijt})^2$ achieves the optimal scenario achievable when only a single currency is used as the trade invoicing currency. Remarkably, the optimal GCP basket then effectively replicates the allocation where there is an independent additional currency used solely for trade invoicing, chosen optimally to minimize misalignment in traded goods prices.

Thus, in the cooperative equilibrium, a well-designed GCP can achieve an equilibrium allocation that reaches the theoretical upper bound, which corresponds to BCP.

One notable feature of Proposition 1 is that it puts a limit on the size of any country's share in the global basket currency. In particular we can state the following:

Corollary 1 *The maximum share that any single country can hold in an optimal global currency basket is* $1/2$*, given cooperative monetary policies are implemented by all countries.*

This follows immediately from the definition of the optimal shares in Proposition 1.

A more important implication of Proposition 1 is that it the optimal currency basket eliminates the need for any country to sacrifice its domestic objectives related to inefficient consumption of domestic non-traded or domestically produced traded goods. In effect there is a conditional "Global Divine Coincidence" in that if the global currency is designed as in Proposition 1, each country can focus solely on its share of the common component of the loss function $\mathcal{L}_i^c$ while at the same time the component related to international trade is minimized subject to the constraints of a basket currency. This suggests an approach to decentralizing the allocation under GCP as follows:

Corollary 2 *If the Global Currency basket is constructed as in Proposition 1, the optimal allocation under BCP can be achieved in a decentralized (non-cooperative) equilibrium if each country* i *chooses its monetary policy to minimize:*

$$\mathbb{E}(\mathcal{L}_i^c) = \mathbb{E}\left(\underbrace{v(m_{it} - z_{it})^2}_{\text{related to } (c_{Nit} - \tilde{c}_{Nit})^2} + \underbrace{(1-v)n_i(m_{it} - z_{it})^2}_{\text{related to } (c_{iit} - \tilde{c}_{iit})^2} \right)$$

where $\mathcal{L}_i^c = v(m_{it} - z_{it})^2 + (1-v)n_i(m_{it} - z_{it})^2$ *represents the part of the common component of the global loss specific to country* i*, with* $\mathcal{L}^c = \sum_{i=1}^N n_i \mathcal{L}_i^c$.

The intuition behind this corollary is clear. If each country minimizes its domestic loss, then the global currency basket will reflect the required response from each country to minimize the losses from international trade. [5]

3.1.1 The two country case

We conduct a detailed analysis of the case where $N = 2$, $v = 0$ and $\sigma_{1z}^2 = \sigma_{2z}^2$ to provide some intuitive insights into how global currency design affects welfare. The size of country 1 is n and its share in global

[5] Note that the condition required for Corollary 2 will in general be binding. As noted below, in general in a non-cooperative game each country will take account of its influence on international trade losses.

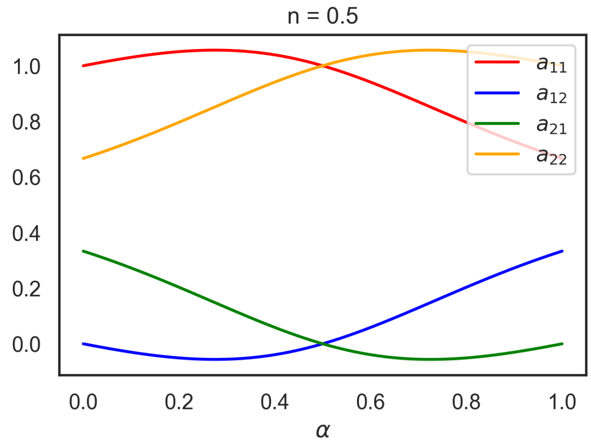

Figure 1: Optimal monetary policy of GCP under cooperative game when $n = 0.5$

Note: The figure 1 shows the response of the optimal monetary policy of when $N = 2, n_1 = n_2 = 0.5, v = 0$, as country 1's global currency share α changes from 0 to 1 under cooperative game.

currency basket is α. The optimal monetary policy $m_{1t} = a_{11}z_{1t} + a_{12}z_{2t}$ and $m_{2t} = a_{21}z_{1t} + a_{22}z_{2t}$ satisfy:

$$a_{11} = \frac{2\alpha^2n^2 - 3\alpha n^2 - 2\alpha n + \alpha + n^2 + n}{\Delta^c}, \qquad a_{12} = \frac{\alpha(2\alpha-1)(n-1)^2}{\Delta^c} = 1 - a_{11};$$
$$a_{21} = \frac{n^2(2\alpha^2 - 3\alpha + 1)}{\Delta^c}, \qquad a_{22} = \frac{2\alpha^2n^2 - 4\alpha^2 n + 2\alpha^2 - \alpha n^2 + n}{\Delta^c} = 1 - a_{21}.$$

where $\Delta^c = 4\alpha^2n^2 - 4\alpha^2n + 2\alpha^2 - 4\alpha n^2 + n^2 + n$. Figure 1 shows a special case for $n = 0.5$. It is evident that the influence of α is not monotonic, a result stemming from the non-linear property of the marginal utility of m_{it}. The non-monotonicity of a_{11} and the occurrences of $a_{11} > 1$ suggest the presence of positive externalities of the global currency in the cooperative game, as m_{1t} would overreact to z_{1t}. When $\alpha = 0.5$, which is the optimal share implied by proposition 1, it is apparent that each country simply targets its domestic productivity shock. The larger is $\alpha > 0.5$, the greater that a country must deviate from this target.

Figure 2 illustrates the role of country size n on the optimal monetary policies for $\alpha = 0.7$. As country 1 grows larger, it targets its own shock exclusively, since trade represents a tiny fraction of the total loss relative to the domestic economy. However, this is not true for the smaller country. For the smaller country, it targets both its own and the larger countries shock in the cooperative equilibrium. While trade overall is small in terms of absolute losses, the smaller country's loss is also small, so an optimal monetary rule for the smaller country still manages its policy to offset the larger country's shock.

Figure 3 illustrates welfare outcomes under alternative shares of the global currency. The red and green line shows the expected losses for country 1 and country 2 when $n = 0.7$, with both countries implementing optimal monetary policies as equation (3.15). In the cooperative game, as country i's share α_i in the global basket increases, the expected loss for country i also increases, meaning that being part of the global currency basket is detrimental to country welfare. The reason is that: in a cooperative equilibrium under GCP, to minimize the foreign tradable consumption deviation $(c_{ijt} - \tilde{c}_{ijt})^2$, m_{gt} should target the optimal Bitcoin

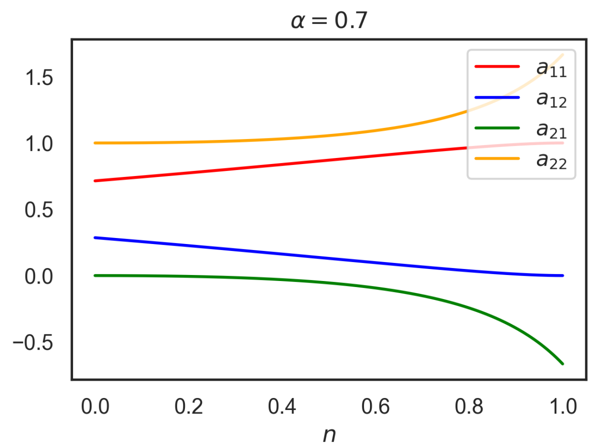

Figure 2: Optimal monetary policy of GCP under cooperative game when $\alpha = 0.7$

Note: The figure 2 shows the response of the optimal monetary policy of when $N = 2, \alpha = 0.7, v = 0$, as country 1's size n changes from 0 to 1 under cooperative game.

supply $m_{bt}^{opt,cB}$. Currencies of countries that become members of the global currency basket would take on the responsibility of guiding the weighted money supply m_{gt} towards $m_{bt}^{opt,cB}$, thus m_{it} must deviate from the domestic consumption target z_{it}.

Figure 3 also shows global welfare for various global currency share, represented by the blue curve. It indicates that the optimal design for the global currency is achieved at $\alpha = 0.5$, while the worst design occurs at $\alpha = 1$, meaning that DCP is the least favourable option under the cooperative equilibrium (if the U.S. is the larger economy).

We then illustrate the impact on country and global welfare under different country sizes n in Figure 4. The red line in the left subplot shows the global currency design that maximizes country 1's welfare for a given country size n under a cooperative game. The results reveal that a country achieves the highest welfare when $\alpha = 0$, while participating in the global currency basket is detrimental to a country's welfare.

In the right subplot, we illustrate the best and worst global currency design that maximizes and minimize the expected global welfare respectively. Just as in Proposition 1, the global currency design that maximizes global welfare involves both countries having equal participation in the global currency basket, i.e. $\alpha = 0.5$, thereby allowing the domestic consumption c_{Nit} and c_{iit} for both countries to reach the efficient level. In contrast, the dark red line demonstrates that the worst global currency design within a two-country model is to assign the global currency role to the larger country. In other words, DCP is the worst global currency design if the U.S. is the larger economy. In a cooperative game, the domestic consumption of the larger country constitutes a significant portion of its overall economy. Consequently, the larger country's monetary policy must consider domestic consumption, reducing its flexibility in influencing international trade.

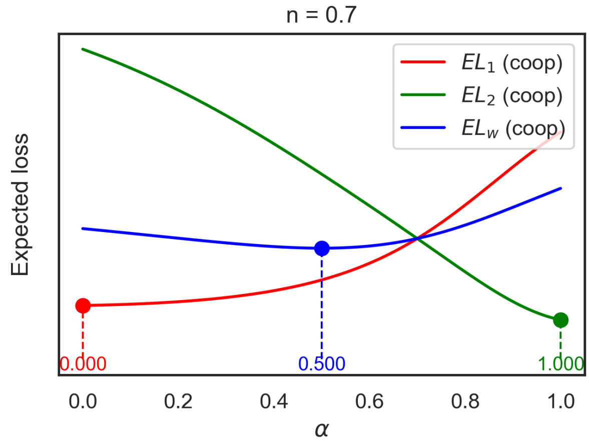

Figure 3: Expected loss of GCP under cooperative game

Note: Figure 3 illustrates the expected loss for country 1, country 2, and global loss under the optimal monetary policy when $N = 2$*,* $n_1 = 0.7, n_2 = 0.3$*,* $v = 0$ *and* $\sigma^2_{1z} = \sigma^2_{2z}$*, as country 1's global currency share* α *varies from 0 to 1 under cooperative game. The red and green lines represent the expected losses for country 1 and country 2, respectively, while the blue line indicates the global loss. The markers on these lines show the values of* α *at which the expected losses reach their minimum.*

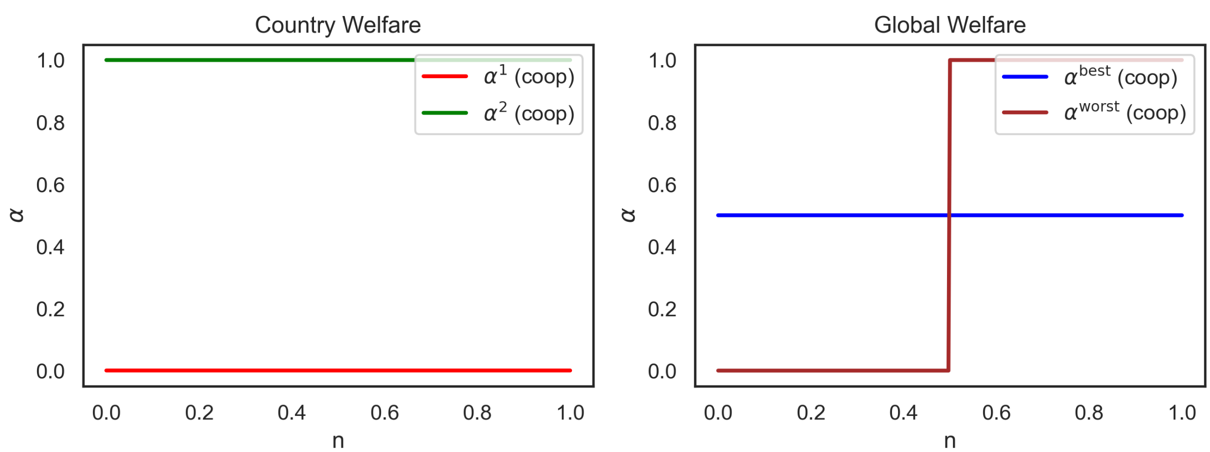

Figure 4: Optimal global currency design under cooperative game

Note: Figure 4 simulates the best and worst global currency design when $N = 2$*,* $v = 0$ *and* $\sigma^2_{1z} = \sigma^2_{2z}$ *under cooperative game. The left subplot shows the global currency design that maximizes each country's welfare, with the red line indicating the value of* α *that maximizes country 1's expected welfare for a given* n*, and the green line representing the design most preferred by country 2. In the right subplot, we depict the global currency designs that yield the maximum and minimum global welfare, with the blue line showing the design that maximizes expected global welfare across different country sizes* n*, and the brown line indicating the design that minimizes global welfare.*

3.2 Noncooperative Monetary Policy

In the Nash equilibrium with commitment, each country's central bank announces m_{it} in advance as a function of shocks, aiming to minimize its expected loss function. We prove in the appendix that the country i's loss function can also be decomposed into three components:

$$\mathbb{E}\left(l_{it}-\tilde{l}_{it}\right)=\delta\ \mathbb{E}\left(v(c_{Nit}-\tilde{c}_{Nit})^2+(1-v)n_i(c_{iit}-\tilde{c}_{iit})^2+(1-v)\sum_{j\neq i}n_j(c_{jit}-\tilde{c}_{jit})^2\right). \tag{3.16}$$

Each country's expected loss function similarly consists of three components: non-tradable consumption $(c_{Nit}-\tilde{c}_{Nit})^2$, domestic tradable consumption $(c_{iit}-\tilde{c}_{iit})^2$, and import consumption deviation $(c_{jit}-\tilde{c}_{jit})^2$. Thus, country i's expected loss function under the four specifications is:

$$PCP:\kappa\ \mathbb{E}\ \left(\mathcal{L}_i^c+(1-v)\sum_{j\neq i}n_j(m_{jt}-z_{jt})^2\right); \tag{3.17}$$

$$LCP:\kappa\ \mathbb{E}\ \left(\mathcal{L}_i^c+(1-v)\sum_{j\neq i}n_j(m_{it}-z_{jt})^2\right); \tag{3.18}$$

$$DCP:\kappa\ \mathbb{E}\ \left(\mathcal{L}_i^c+(1-v)\sum_{j\neq i}n_j(m_{1t}-z_{jt})^2\right); \tag{3.19}$$

$$GCP:\kappa\ \mathbb{E}\ \left(\mathcal{L}_i^c+(1-v)\sum_{j\neq i}n_j(m_{gt}-z_{jt})^2\right). \tag{3.20}$$

where $\mathcal{L}_i^c=v(m_{it}-z_{it})^2+(1-v)n_i(m_{it}-z_{it})^2$, has been defined in Corollary 2, denotes country i's domestic welfare loss from $(c_{Nit}-\tilde{c}_{Nit})^2$ and $(c_{iit}-\tilde{c}_{iit})^2$.

Compared to equations (3.2) to (3.5), in the Nash case, country i disregards two key considerations under cooperation: first, the externalities of monetary policy, as country i does not account for the impact of m_{it} on other countries' consumption, i.e. $(c_{ijt}-\tilde{c}_{ijt})^2$; and second, its relative share in global welfare is n_i, but this is overlooked in the Nash equilibrium.

We now analyze the optimal monetary policy under each pricing paradigm and compare them to the outcome under cooperation.

Producer currency pricing. From equation (3.17), it can be observed that country i's monetary policy can only influence its' domestic loss $\mathcal{L}_i^c$, while it lacks the capacity to affect country i's imports consumption deviation $(c_{jit}-\tilde{c}_{jit})^2$. The optimal allocation for domestic consumption requires country i's monetary policy to target z_{it}, resulting in the following optimal policy:

$$m_{it}^{opt,nP}=\frac{v}{\Delta_i^{nP}}z_{it}+\frac{(1-v)n_i}{\Delta_i^{nP}}z_{it}=z_{it},$$

where the superscript "nP" represents "Nash game" and "PCP", and $\Delta_i^{nP}=v+(1-v)n_i$ is the sum of the weights. This leads to the same optimal policy as in the cooperative case. Moreover, it is evident that

$m_{it} = z_{it}$ is not only a Nash equilibrium but also a dominant strategy equilibrium. [6]

Local currency pricing. From equation (3.18), we can see that the optimal monetary policy under LCP in the Nash equilibrium is exactly the same as in the cooperative case, as shown in equation (3.8). Additionally, there are no monetary policy externalities under LCP, since the central bank i fully considers the impact of m_{it} on foreign imports $(c_{ijt} - \tilde{c}_{ijt})^2$.

Dollar currency pricing. In DCP, country 1 and the other countries are asymmetric. For non-U.S. country i, welfare loss from import consumption $(c_{jit} - \tilde{c}_{jit})^2$, is beyond the influence of country i's policy. Therefore, just as in the cooperative problem, i.e. equation (3.10), country i's optimal monetary policy is $m_{it}^{opt,nD} = z_{it}$ for $i \neq 1$, ensuring domestic loss $\mathcal{L}_i^c$ reach the efficient level.

Compared to the cooperative equilibrium, country 1's monetary policy in the uncooperative equilibrium only takes into account of three components: ① country 1's non-tradable goods $(c_{N1t} - \tilde{c}_{N1t})^2$, ② country 1's domestic tradable goods $(c_{11t} - \tilde{c}_{11t})^2$, and ③ country 1's imports $(c_{i1t} - \tilde{c}_{i1t})^2$ for $i \neq 1$. The first two require m_{1t} to target z_{1t}, while the last one requires it to target z_{it}. Thus, country 1's optimal uncooperative monetary policy is given by:

$$m_{1t}^{opt,nD} = \frac{v}{\Delta_1^{nD}} z_{1t} + \frac{(1-v)n_1}{\Delta_1^{nD}} z_{1t} + (1-v) \sum_{j \neq 1} \left(\frac{n_j}{\Delta_1^{nD}} z_{jt} \right), \tag{3.21}$$

where $\Delta_1^{nD} = v + (1-v)n_1 + (1-v)\Sigma_{j\neq 1} n_j$ is the sum of weights. When $v = 0, N = 2$, equation (3.21) is same as the optimal monetary policy in Devereux et al. (2007). Furthermore, for country 1, equation (3.21) is not only a Nash equilibrium but also a dominant strategy, which implies the potential lack of credibility in cooperative commitments.

By comparing equation (3.11) and equation (3.21), we observe that country 1's monetary policy m_{1t} in the uncooperative equilibrium deviates in two significant ways compared with the cooperative equilibrium. First, in the uncooperative equilibrium, country 1 does not consider how its monetary policy affects other countries' consumption through its exports, thereby ignoring the impact of m_{1t} on $(c_{1jt} - \tilde{c}_{1jt})^2$. Second, the weights assigned to productivity shocks across countries are not optimal. In equation (3.11), country 1 would assign a weight of $\frac{n_j(1-n_j)}{\Delta_1^{cD}}$ to the productivity shock z_{jt} of country j. However, in the uncooperative outcome, this weight is reduced to n_j, indicating that country 1 only considers the impact of other countries' productivity on its own import consumption, while neglecting the share of each country's welfare in the overall global welfare.

[6] Although it may seem that both the cooperative and uncooperative outcomes under PCP implement the same monetary policy $m_{it} = z_{it}$ to achieve the same allocation, this is merely a coincidence. In fact, under PCP, the uncooperative problem involves monetary policy externalities. As seen from equation (3.6), m_{it} affects foreign imports $(c_{ijt} - \tilde{c}_{ijt})^2$, but in the uncooperative game, the central bank i does not consider this because it is home imports $(c_{jit} - \tilde{c}_{jit})^2$, rather than home exports $(c_{ijt} - \tilde{c}_{ijt})^2$, that enter country i's utility function. For example, if a country's non-tradable goods shock and tradable goods shock were different, i.e., $z_{it}^N \neq z_{it}$, then the optimal monetary policy of PCP under the uncooperative outcome would differ from that in the cooperative outcome.

Bitcoin currency pricing. The Bitcoin measure in the non-cooperative game must be defined separately for each country. That is defined as the optimal policy for each country if it had control of a currency solely used for international trade invoicing. The loss function in this case for country i would be

$$BCP : \kappa \, \mathbb{E} \left(\mathcal{L}_i^c + (1-v) \sum_{j \neq i} n_j (m_{bt} - z_{jt})^2 \right). \tag{3.22}$$

The Bitcoin issuer, aim to maximize the country i's welfare, will not take country i's productivity shock z_{it} into account when determining the Bitcoin supply, as it does not affect country i's imported consumption $(c_{jit} - \tilde{c}_{jit})^2$. The optimal monetary policy set by the hypothetical Bitcoin issuer in country i would be:

$$m_{bit}^{opt,nB} = \frac{1}{1-n_i} \sum_{j \neq i} (n_j z_{jt}), \tag{3.23}$$

where $m_{bit}^{opt,nB}$ represents the optimal Bitcoin supply if Bitcoin is issued by country i. Mathematically, $m_{bit}^{opt,nB}$, reflects the optimal supply of the pricing currency that maximize country i's welfare from import consumption $(c_{jit} - \tilde{c}_{jit})^2$ when there is only one currency used for international trade invoicing.

Global currency pricing. As shown in equation (3.20), under GCP, if country i is included in the global currency basket, its monetary policy can influence three terms: ① country i's non-tradable goods $(c_{Nit} - \tilde{c}_{Nit})^2$, ② country i's domestic tradable goods $(c_{iit} - \tilde{c}_{iit})^2$, and ③ country i's import consumption $(c_{jit} - \tilde{c}_{jit})^2$. Compared to the cooperative game, country i doesn't consider the impact of m_{it} on its export.

The optimal monetary policy for country i under GCP is as follows:

$$\begin{aligned} m_{it}^{opt,nG} &= z_{it} + \frac{\alpha_i(1-v)(1-n_i)}{v+(1-v)n_i} \left(m_{bit}^{opt,nB} - m_{gt}^{opt,nG} \right), \\ m_{gt}^{opt,nG} &= \frac{\sum_{i=1}^N \left(\alpha_i z_{it} + \frac{\alpha_i^2(1-v)(1-n_i)}{v+(1-v)n_i} m_{bit}^{opt,nB} \right)}{1 + \sum_{i=1}^N \left(\frac{\alpha_i^2(1-v)(1-n_i)}{v+(1-v)n_i} \right)}. \end{aligned} \tag{3.24}$$

Equation (3.24) indicates that country i's monetary policy m_{it} aims to balance two objectives: the first is targeting z_{it} to ensure that domestic loss, $\mathcal{L}_i^c$, reaches the efficient level; the second is to adjust the global currency supply m_{gt} to be closer to $m_{bit}^{opt,nB}$ in order to optimize country i's import consumption $(c_{jit} - \tilde{c}_{jit})^2$. Notice that, in contrast to the cooperative game, different countries have distinct goals since $m_{bit}^{opt,nB} \neq m_{bjt}^{opt,nB}$ if $i \neq j$, so country i's preference for global currency supply is different as shown in equation (3.24).

Therefore, the positive externality of the global currency that we proposed in the cooperative game does not hold in the uncooperative equilibrium. In the cooperative equilibrium, all countries share a joint trade allocation goal, i.e. $m_{bt}^{opt,cB}$. This positive externality leads to a free-rider phenomenon where currencies in the global currency basket must make sacrifices, while those outside can free-ride. However, the scenario changes entirely when strategic responses between players are considered, so there is **negative externality of the global currency under the uncooperative game**: each country pursues its own trade allocation goal, $m_{bit}^{opt,nB}$, which varies across countries. The inconsistency in each country's policy goal $m_{bit}^{opt,nB}$ results in negative externalities when they implement their monetary policies since countries neglect the effects of their monetary policies on exports.

Optimal global currency design. It is difficult to derive an analytical solution for the optimal global currency design in the Nash equilibrium, but we can offer some intuition to show that it is fundamentally different from the cooperative equilibrium in a two-country model.

3.2.1 The two country case

Focus now on the uncooperative outcome when $N = 2$, $v = 0$ and $\sigma_{1z}^2 = \sigma_{2z}^2$. Assuming the optimal monetary policy is $m_{1t} = a_{11}z_{1t} + a_{12}z_{2t}$ and $m_{2t} = a_{21}z_{1t} + a_{22}z_{2t}$, the solutions for $(a_{11}, a_{12}, a_{21}, a_{22})$ under Nash equilibrium are given by:

$$a_{11} = \frac{n(1-\alpha-\alpha n-\alpha^2 n+\alpha^3 n+2\alpha^2-\alpha^3)}{\Delta^n}, \qquad a_{12} = \frac{\alpha(1-n)(\alpha+n-3\alpha n+\alpha^2 n)}{\Delta^n} = 1-a_{11};$$
$$a_{21} = \frac{n(1-\alpha)(n-\alpha n-\alpha^2 n+\alpha^2)}{\Delta^n}, \qquad a_{22} = \frac{(1-n)(n-2\alpha^2 n+\alpha^3 n+\alpha^2)}{\Delta^n} = 1-a_{21}.$$

where $\Delta^n = 2\alpha^2 n^2 - 2\alpha^2 n + \alpha^2 - 2\alpha n^2 + n$. Figure 5 illustrates the monetary policy when $n = 0.5$. It is evident that these parameters are monotonic and always lie between 0 and 1. A higher α provides country 1 with greater leverage to influence the global currency supply m_{gt}. Thus, as shown in equation (3.24), country 1 will place more emphasis on import consumption $(c_{21t} - \tilde{c}_{21t})^2$, leading to a larger deviation from domestic consumption goal, i.e. a larger deviation from z_{1t}. Consequently, a_{11} decreases and a_{12} increases.

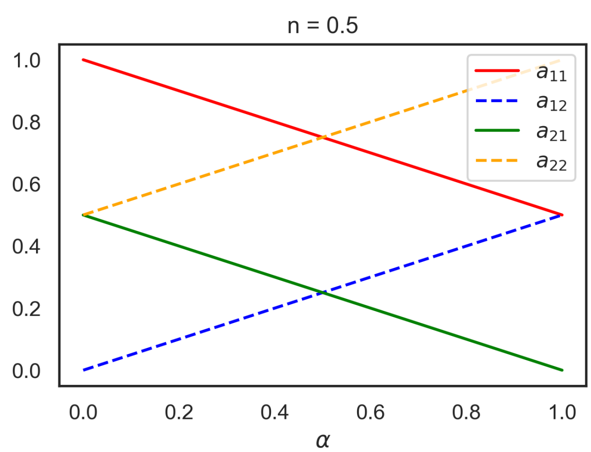

Figure 5: Optimal monetary policy of GCP under Nash game

Note: The figure 5 shows the response of the optimal monetary policy of when $N = 2, n_1 = n_2 = 0.5, v = 0$ and $\sigma_{1z}^2 = \sigma_{2z}^2$, as country 1's global currency share α changes from 0 to 1 under Nash game.

In the two-country model, the optimal global currency supply m_{gt} preferred by country 1 and country 2 satisfies: $m_{b1t}^{opt,nB} = z_{2t}$ and $m_{b2t}^{opt,nB} = z_{1t}$, respectively. This indicates that country 1 prefers the global currency to fully respond to country 2's productivity shock while ignoring its own. Conversely, country 2 aims for the opposite, wanting the global currency to react entirely to country 1's productivity shock. Therefore, countries with a larger share in the global currency basket will have its monetary policy m_{it} responds more to foreign shocks, enhancing its import consumption welfare $(c_{jit} - \tilde{c}_{jit})^2$. This also explains why the blue dashed line in Figure 5 slopes upwards.

Figure 6 shows the expected losses for country 1, country 2, and the global economy under the optimal monetary policy when $n = 0.7$. The red line illustrates how the expected loss of the larger country (country 1) varies with the global currency basket share α. The U-shape of the graph indicates that country 1's welfare reaches its maximum at $\alpha = 0.403$. This is because as country 1's share in the global currency basket increases, i.e., $\alpha \uparrow$, its ability to influence the global currency also increases. Then, country 1's monetary policy would move away from targeting domestic consumption z_{1t} towards targeting import consumption $m_{b1t}^{opt,nB} = z_{2t}$, as shown in Figure 5. In this process, two effects that impact country 1's welfare occurs: one is the rise in domestic consumption deviations $(c_{N1t} - \tilde{c}_{N1t})^2$ and $(c_{11t} - \tilde{c}_{11t})^2$ because the monetary policy moves away from z_{1t}; the other is the potential decline in import consumption deviation $(c_{21t} - \tilde{c}_{21t})^2$ as the monetary policy converges more closely to $m_{b1t}^{opt,nB} = z_{2t}$. The interplay of these two effects causes country 1's expected loss to exhibit a U-shaped pattern.

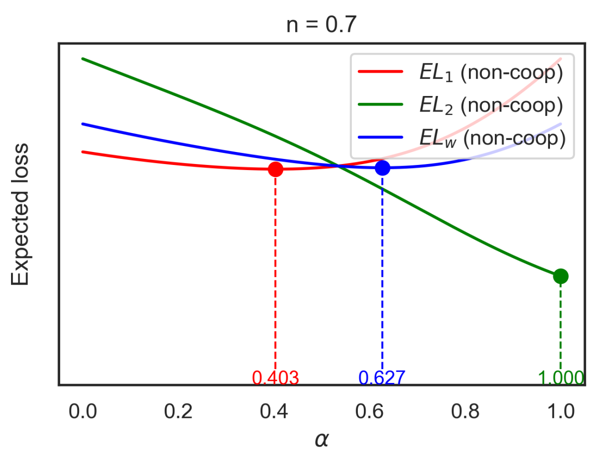

Figure 6: Expected loss of GCP under Nash game

Note: Figure 6 illustrates the expected loss for country 1, country 2, and global loss under the optimal monetary policy when $N = 2$, $n_1 = 0.7, n_2 = 0.3$, $v = 0$ and $\sigma_{1z}^2 = \sigma_{2z}^2$, as country 1's global currency share α varies from 0 to 1 under Nash game. The red and green lines represent the expected losses for country 1 and country 2, respectively, while the blue line indicates the global loss. The markers on these lines show the values of α at which the expected losses reach their minimum.

The green line in Figure 6 shows the smaller country's (country 2) expected loss, which decreases monotonically. As country 1's share in the global currency basket α increases, country 2 loses its influence over the global currency supply. Consequently, its monetary policy becomes more focused on domestic consumption $(c_{N2t} - \tilde{c}_{N2t})^2$ and $(c_{22t} - \tilde{c}_{22t})^2$, aligning more closely with z_{2t} and diverging from $m_{b2t}^{opt,nB} = z_{1t}$, as in Figure 5. In this process, two effects occur too: one is the reduction in domestic consumption deviations

$(c_{N2t} - \tilde{c}_{N2t})^2$ and $(c_{22t} - \tilde{c}_{22t})^2$, as monetary policy coverages to z_{2t}; the other is the potential increase in import consumption deviation $(c_{12t} - \tilde{c}_{12t})^2$, as monetary policy moves away from $m_{b2t}^{opt,nB} = z_{1t}$. The simulation results indicate that the first effect dominates, leading country 2 to prefer a lower global basket share.

Figure 6 also displays the global loss as the blue line for $n = 0.7$, indicating that the economy reaches its optimal allocation at $\alpha = 0.627$ and the worst allocation at $\alpha = 0$. This suggests that the optimal global currency design should assign a bigger weight to the larger economy rather than placing the responsibility of the global currency on the smaller economy. This is because larger economies i can more easily influence international trade, thereby moving towards their preferred $m_{bit}^{opt,nB}$ with a relatively small sacrifice in domestic consumption. In contrast, if a smaller economy j holds a larger share of the global currency, it incurs a higher cost to skew trade allocation toward $m_{bjt}^{opt,nB}$. Therefore, an effective global currency configuration should grant a larger share to the larger economy, whereas the worst design would place the global currency burden on the smaller country.

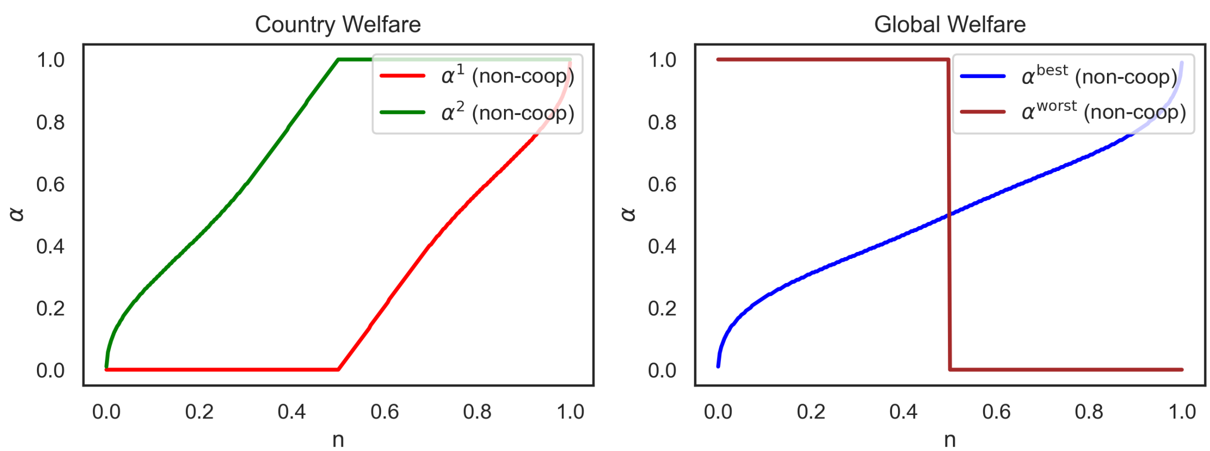

Figure 7: Optimal global currency design under the Nash game

Note: Figure 7 simulates the best and worst global currency design when $N = 2$, $v = 0$ and $\sigma_{1z}^2 = \sigma_{2z}^2$ under Nash game. The left subplot shows the global currency design that maximizes each country's welfare, with the red line indicating the value of α that maximizes country 1's expected welfare for a given n, and the green line representing the design most preferred by country 2. In the right subplot, we depict the global currency designs that yield the maximum and minimum global welfare, with the blue line showing the design that maximizes expected global welfare across different country sizes n, and the brown line indicating the design that minimizes global welfare.

Figure 7 examines how country welfare and global welfare behave for any country size n. The left panel shows the values of α that maximize the welfare of country 1 and country 2 for different country sizes n. It can be observed that larger economies prefer their own currency to hold a certain share, while smaller economies are reluctant to join the global currency basket. This is totally different from the cooperative game. In the cooperative game, countries have an incentive to avoid becoming the global currency because the objective of all countries is same – whoever takes on this responsibility will aim to adjust the global currency towards $m_{bt}^{opt,cB}$. Therefore, the country that takes on the global currency role will be burdened by others acting as free riders. In contrast, in the Nash game, the objectives of countries are conflicting, creating an incentive for the larger country to have its currency hold a certain share in the global currency basket because it can easily influence trade. Smaller countries, on the other hand, are unable to take on the responsibility of the global currency because it would significantly compromise their domestic consumption.

The right panel of Figure 7 shows the global currency designs that lead to the highest and lowest global welfare, to ensure the robustness of Figure 6 for all country size n. It shows that a good design allocates a larger share of the global currency to the major economy, while a poor design places the global currency responsibility on the smaller country. This is also in stark contrast to the cooperative game, where an equal contribution from both countries to the global currency would be optimal, while having the larger country bear the global currency responsibility represents the worst design. Thus, the strategic framework of monetary policy significantly influences the optimal composition of the global currency basket.

3.3 Policy implementation

Due to the static nature of the model, the implementation of monetary policy is characterized as a money supply rule. In standard New Keynesian models, we are used to thinking of policy as a target rule for stabilizing an inflation index. In the optimal policy rules, outlined above in both Cooperative and Non-cooperative cases is isomorphic to a rule which stabilizes different price indices. Table 1 outlines the different indices corresponding to optimal policies under different pricing strategies. For instance, in the cooperative or Nash case under PCP, the optimal rule stabilizes the Producer Price Index (PPI), while under LCP it is the Consumer Price Index (CPI) that is stabilized. Under DCP and GCP, the price index generally differs between cooperative and Nash cases, and is not described either by the PPI or CPI. In the dynamic model described below, an optimal monetary policy targets and inflation rate rather than a price index. The Appendix provides further description of this Table.

Table 1: The price targeting rule of country i under various pricing paradigms, $i \in \{1, 2, ..., N\}$

Pricing Paradigm	Cooperative Game	Nash game
PCP	PPI	PPI
LCP	CPI	CPI
DCP	The currency i price of all goods priced in currency i and consumed **globally**	The currency i price of all goods priced in currency i and consumed **by country** i
BCP	The currency i price of all goods priced in currency i	The currency i price of all goods priced in currency i
GCP	The currency i price of all goods priced in currency $i + \alpha_i \times$ the global currency price of all goods priced in global currency and consumed **globally**	The currency i price of all goods priced in currency $i + \alpha_i \times$ the global currency price of all goods priced in global currency and consumed **by country** i

4 Extensions: Monetary Shocks

We now consider a scenario where each country faces additional exogenous shocks to its monetary policy, denoted as $\boldsymbol{\mu}_t = (\mu_{1t}, \cdots, \mu_{Nt})'$, such as velocity shocks, which are beyond the control of the monetary authority. The central bank in country i commits to implement the monetary policy equation $m_{it} = a_{i1}z_{1t} + \ldots + a_{iN}z_{1N}$, but the actual monetary supply turns out to be:

$$m_{it} = a_{i1}z_{1t} + \ldots + a_{iN}z_{1N} + \mu_{it}, \tag{4.1}$$

where μ_{it} represents an unexpected monetary shock in country i. Central banks choose monetary policy parameters $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$ at the beginning of each period and adhere to this commitment after all shocks occur, but they cannot respond to $\boldsymbol{\mu}_t$. The velocity shocks introduce a level of uncertainty, highlighting the central bank's inability to fully control the monetary supply.

We further assume that productivity shocks $\boldsymbol{z}_t = (z_{1t}, \cdots, z_{Nt})'$ and velocity shocks $\boldsymbol{\mu}_t = (\mu_{1t}, \cdots, \mu_{Nt})'$ are orthogonal both internally and between each other, with variances given by $\boldsymbol{\sigma}_z^2 = (\sigma_{1z}^2, \cdots, \sigma_{Nz}^2)'$ and $\boldsymbol{\sigma}_\mu^2 = (\sigma_{1\mu}^2, \cdots, \sigma_{N\mu}^2)'$.

4.1 Global Currency Pricing

We first consider the case of GCP, where international trade uses a global currency for invoicing. The central bank in country i can select $(a_{i1}, \ldots, a_{iN})$ in equation (4.1). Thus, the global currency supply m_{gt} is given by

$$m_{gt} = \sum_{i=1}^N \sum_{j=1}^N \alpha_i a_{ij} z_{jt} + \sum_{i=1}^N \alpha_i \mu_{it}.$$

Since the global currency is a composite of N fiat currencies, its supply is influenced by global monetary shocks $\boldsymbol{\mu}_t$, weighted by each country's share α_i in the global currency basket. Unlike single-currency pricing paradigms such as PCP, LCP, and DCP, where exports from one country to another are impacted by the monetary policies of individual countries, GCP offers a stability advantage due to its diversified structure.

4.1.1 Global Loss Function

When considering unresponsive monetary shocks, there are two types of distortions in the economy: one arises from currency misalignment due to global currency pricing, and the other results from uncontrollable monetary volatility that prevent monetary policy from fully responding to productivity shocks. The orthogonality between the shocks allows us to easily decompose the global loss function into two parts for any monetary policy rules $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$ and any global currency design $\boldsymbol{\alpha}$:

$$L^G = \underbrace{\boldsymbol{\chi}_z^G(v, \boldsymbol{n}, \boldsymbol{\alpha}, \boldsymbol{a})'\boldsymbol{\sigma}_z^2}_{\text{currency misalignment}} + \underbrace{\boldsymbol{\chi}_\mu^G(v, \boldsymbol{n}, \boldsymbol{\alpha})'\boldsymbol{\sigma}_\mu^2}_{\text{monetary volatility}}, \tag{4.2}$$

where the column vectors $\boldsymbol{\chi}_z^G$ and $\boldsymbol{\chi}_\mu^G$, functions of $(v, \boldsymbol{n}, \boldsymbol{\alpha}, \boldsymbol{a})$ and $(v, \boldsymbol{n}, \boldsymbol{\alpha})$ respectively, describe the impact of productivity and velocity shocks on welfare. Global losses stem from two main distortions, currency misalignment and monetary volatility. The monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$ can only mitigate currency misalignment but cannot address monetary volatility, as the velocity shocks $\boldsymbol{\mu}_t$ are unresponsive in the setup.

Currency misalignment. Note that in equation (4.2), the selection of the monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$ impacts only $\boldsymbol{\chi}_z^G$ and does not affect $\boldsymbol{\chi}_\mu^G$. When committing to a monetary policy, the government does not need to account for potential future monetary shocks $\boldsymbol{\sigma}_\mu^2$. Thus, when countries implement cooperative monetary policies to maximize global welfare, the money supply is same as the baseline model but now incorporating the velocity shock u_{it}, slightly rearranged below:

$$
\begin{gathered}
m_{it}^{opt,cG} = z_{it} - \frac{(1-v)\alpha_i}{vn_i + (1-v)n_i^2}\, x_t + u_{it}, \\
x_t = \frac{\sum_{i=1}^N \left(n_i(1-n_i)\left(\sum_{j=1}^N(\alpha_j z_{jt}) - z_{it}\right)\right)}{1 + (1-v)\left(\sum_{i=1}^N n_i(1-n_i)\right)\left(\sum_{i=1}^N\left(\frac{\alpha_i^2}{vn_i + (1-v)n_i^2}\right)\right)}.
\end{gathered}
\tag{4.3}
$$

Due to the separability of productivity shocks and monetary shocks as shown in equation (4.2), the analysis concerning $\boldsymbol{\chi}_z^G(v, \boldsymbol{n}, \boldsymbol{\alpha}, \boldsymbol{a})'\boldsymbol{\sigma}_z^2$ is identical to the baseline model. Monetary authorities continue to follow optimal policies under GCP to offset domestic and international productivity shocks.

Monetary volatility. We provide the expression for the welfare loss from monetary volatility $\boldsymbol{\chi}_\mu^G(v, \boldsymbol{n}, \boldsymbol{\alpha})'\boldsymbol{\sigma}_\mu^2$ in equation (4.2):

$$
\boldsymbol{\chi}_\mu^G(v, \boldsymbol{n}, \boldsymbol{\alpha})'\boldsymbol{\sigma}_\mu^2 = v\left(\sum_{i=1}^N n_i\sigma_{iu}^2\right) + (1-v)\left(\sum_{i=1}^N n_i^2\sigma_{iu}^2\right) + (1-v)\left(\sum_{i=1}^N n_i(1-n_i)\right)\left(\sum_{i=1}^N \alpha_i^2\sigma_{iu}^2\right). \tag{4.4}
$$

The negative effects of uncontrollable velocity shocks also consist of three parts: deviations in non-tradable consumption, domestic tradable consumption, and international trade.

The final/third term in the RHS of equation (4.4) highlights why GCP differs from other single-currency pricing paradigms like PCP, LCP, and DCP. Because GCP is composed of multiple currencies, it can significantly mitigate the negative effects of monetary volatility in international trade. The impact of monetary shocks on international trade, represented as $\sum_{i=1}^N \alpha_i^2\sigma_{iu}^2$, depends on the share of each currency in the basket. The global currency gains stability from being composed of a diverse basket of currencies.

4.1.2 Optimal Global Currency Design

The global loss function, as outlined in equation (4.2), demonstrates that the design of the global currency, $\boldsymbol{\alpha}$, influences welfare through two channels: currency misalignment and monetary volatility. Based on this, we propose the following:

Proposition 2 *For any country sizes set $\boldsymbol{n}$, variance set $(\boldsymbol{\sigma}_z^2, \boldsymbol{\sigma}_\mu^2)$, we find that:*

(1) Suppose the government commit to the optimal cooperative monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$ as equation (4.3). The optimal global currency design $\boldsymbol{\alpha}$ to minimize currency misalignment, as defined by

$\boldsymbol{\chi}_z^G(v,\boldsymbol{n},\boldsymbol{\alpha},\boldsymbol{a})$, is given by:

$$\alpha_i = \frac{n_i(1-n_i)}{\sum_{j=1}^N n_j(1-n_j)}.$$

(2) For any monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$, the optimal global currency design $\boldsymbol{\alpha}$ to minimize monetary volatility, as captured by $\boldsymbol{\chi}_\mu^G(v,\boldsymbol{n},\boldsymbol{\alpha})$, is given by:

$$\alpha_i = \frac{1/\sigma_{i\mu}^2}{\sum_{j=1}^N (1/\sigma_{j\mu}^2)}.$$

The relative volatility of productivity shocks $\boldsymbol{\sigma}_z^2$ and monetary shocks $\boldsymbol{\sigma}_\mu^2$ determines how the optimal global currency design $\boldsymbol{\alpha}$ should be structured.

This theorem extends Proposition 1 in the baseline model. It illustrates that when exists other non-responsive shocks $\boldsymbol{\mu}_t$, the optimal design of a global currency $\boldsymbol{\alpha}$ needs to account for the variability of velocity shocks across countries. We can improve welfare by giving more stable currencies a larger share in the global currency basket.

4.2 Welfare Ranking

In this section, we compare how welfare performance varies under optimal cooperative monetary policy across different pricing paradigms: PCP, LCP, DCP, and GCP. This means that in the pricing paradigm X, where $X \in \{P, L, D, G\}$, the monetary policy adheres to the optimal strategy defined for that paradigm, similar to the baseline model, but with the inclusion of the monetary shock μ_{it}. For simplicity, we assume that the shock variances are the same across all countries, meaning $\sigma_{iz}^2 \equiv \sigma_z^2$ and $\sigma_{i\mu}^2 \equiv \sigma_\mu^2$. In the pricing paradigm X, the global loss function can also be decomposed into two parts:

$$L^{cX}(v,\boldsymbol{n}) = \underbrace{\chi_z^{cX}(v,\boldsymbol{n})\sigma_z^2}_{\text{currency misalignment}} + \underbrace{\chi_\mu^{cX}(v,\boldsymbol{n})\sigma_\mu^2}_{\text{monetary volatility}}, \qquad X \in \{P, L, D, G\},$$

where $\chi_z^{cX}(v,\boldsymbol{n})$ and $\chi_\mu^{cX}(v,\boldsymbol{n})$ represent the losses from currency misalignment and monetary volatility under a cooperative policy within pricing paradigm X, respectively, while $L^{cX}(v,\boldsymbol{n})$ represents the total loss. There are the following differences in welfare outcomes across the four pricing paradigms:

Proposition 3 *If countries implement a cooperative monetary policy, then for any set of country sizes $\boldsymbol{n}$, we have:*

(1) In terms of reducing currency misalignment, we find that $PCP \succeq GCP \succeq DCP \succeq LCP$, with global currency designed as $\alpha_i^* = n_i(1-n_i)/\sum_{j=1}^N n_j(1-n_j)$ under GCP. Mathematically, it means:

$$0 = \chi_z^{cP}(v,\boldsymbol{n}) \le \chi_z^{cG}(v,\boldsymbol{n},\boldsymbol{\alpha})|_{\alpha_i=\alpha_i^*} \le \chi_z^{cD}(v,\boldsymbol{n}) \le \chi_z^{cL}(v,\boldsymbol{n}).$$

(2) In terms of reducing monetary volatility, we find that $GCP \succeq PCP = DCP = LCP$. Mathematically, it means for any global currency design $\boldsymbol{\alpha}$:

$$0 \le \chi_\mu^{cG}(v,\boldsymbol{n},\boldsymbol{\alpha}) \le \chi_\mu^{cP}(v,\boldsymbol{n}) = \chi_\mu^{cD}(v,\boldsymbol{n}) = \chi_\mu^{cL}(v,\boldsymbol{n}).$$

The expressions for $\chi_z^{cX}(v, \boldsymbol{n})$ and $\chi_\mu^{cX}(v, \boldsymbol{n})$ are provided in the appendix. The proposition shows that PCP effectively addresses currency misalignment, whereas GCP excels at reducing the negative impacts of monetary volatility. DCP and LCP, however, do not offer significant benefits in either aspect. Next, we give a detailed explanation about welfare ranking.

Currency misalignment. In terms of reducing currency misalignment, we have

$$PCP \succeq GCP(BCP) \succeq DCP \succeq LCP.$$

Since we only consider the special case of $\alpha_i = n_i(1-n_i)/\sum_{j=1}^{N} n_j(1-n_j)$, GCP is same as BCP if there are no velocity shocks.

Firstly, in term of currency misalignment, PCP is superior to BCP, and BCP is, in turn, better than DCP. PCP ensures efficiency across all three consumption categories: non-tradable goods $(c_{Nit} - \tilde{c}_{Nit})^2$, domestic tradable goods $(c_{iit} - \tilde{c}_{iit})^2$, and foreign tradable goods $(c_{jit} - \tilde{c}_{jit})^2$. BCP, while able to achieve efficiency in non-tradable and domestic tradable goods, falls short in ensuring efficient international trade $(c_{jit} - \tilde{c}_{jit})^2$. DCP, however, incurs even greater welfare losses than BCP, as it fails to deliver efficiency in any of three consumption categories.

Secondly, BCP, GCP, and DCP all outperform LCP under the cooperative game. In BCP/GCP/DCP, only a single currency is used for pricing. As a result, achieving an efficient tradable consumption allocation requires $m_{bt}/m_{gt}/m_{1t}$ to target the productivity shocks z_{it} for $\forall i$, as shown in equation (3.9), (3.12) and (3.14). However, under LCP, the monetary policy of any country i, m_{it} for $\forall i$, must target the productivity shocks z_{jt} for $\forall j$, as in equation (3.7). This leads to even greater currency misalignment in LCP.

Monetary volatility. In terms of reducing monetary volatility, we have

$$GCP \succeq PCP = DCP = LCP.$$

GCP stands out from the other three paradigms because it uses a composite currency. In PCP, LCP, and DCP, imports from one country to another are priced in just one currency, which inevitably absorbs the monetary shocks of that currency. In contrast, GCP employs a basket of currencies for pricing in international trade, which significantly reduces the overall currency volatility of the basket. This gives GCP an advantage in reducing the impact of monetary fluctuations in international trade.

PCP and GCP each have advantages in mitigating productivity and velocity shocks respectively, thus we can make the following corollary:

Corollary 3 *Suppose $\alpha_i^* = n_i(1-n_i)/\sum_{j=1}^{N} n_j(1-n_j)$, for any set of country sizes $\boldsymbol{n}$, there exists a cut-off point that satisfies:*

(1) if $\sigma_u^2/\sigma_z^2 < 1$, in term of global welfare, we have $PCP \succeq GCP \succeq DCP \succeq LCP$. Mathematically, it means:

$$L^{cP}(v, \boldsymbol{n}) \leq L^{cG}(v, \boldsymbol{n}, \boldsymbol{\alpha})|_{\alpha_i = \alpha_i^*} \leq L^{cD}(v, \boldsymbol{n}) \leq L^{cL}(v, \boldsymbol{n}).$$

(2) if $\sigma_u^2/\sigma_z^2 > 1$, in term of global welfare, we have $GCP \succeq PCP \succeq DCP \succeq LCP$. Mathematically, it means:

$$L^{cG}(v, \boldsymbol{n}, \boldsymbol{\alpha})|_{\alpha_i=\alpha_i^*} \leq L^{cP}(v, \boldsymbol{n}) \leq L^{cD}(v, \boldsymbol{n}) \leq L^{cL}(v, \boldsymbol{n}).$$

Thus, $\sigma_u^2/\sigma_z^2 = 1$ serves as a cut-off point that determines whether PCP or GCP is more advantageous in achieving greater global welfare. Greater volatility in productivity shocks favors PCP, while greater volatility in monetary shocks benefits GCP.

5 Dynamic Model

The baseline model uses one-period Calvo pricing to simplify monetary policy trade-offs, reducing it to a static framework. To capture dynamics with staggered prices, we extend it to dynamic Calvo pricing as in Clarida et al. (2002). The definition of equilibrium remains the same, except that firm's price-setting strategy changes to (e.g., for GCP):

$$\tilde{P}_{jjt} = \frac{E_t\left[\sum_{s=0}^{\infty}\theta^s Q_{jt,t+s} MC_{j,t+s}(P_{jj,t+s})^{\varepsilon} Y_{jj,t+s}\right]}{E_t\left[\sum_{s=0}^{\infty}\theta^s Q_{jt,t+s}(P_{jj,t+s})^{\varepsilon} Y_{jj,t+s}\right]},$$

$$\tilde{P}_{j-jt}^G = \frac{E_t\left[\sum_{s=0}^{\infty}\theta^s Q_{jt,t+s} MC_{j,t+s}(P_{j-j,t+s}^G)^{\varepsilon}\left(\sum_{i\neq j} n_i Y_{ji,t+s}\right)\right]}{E_t\left[\sum_{s=0}^{\infty}\theta^s Q_{jt,t+s}\mathcal{E}_{jg,t+s}(P_{j-j,t+s}^G)^{\varepsilon}\left(\sum_{i\neq j} n_i Y_{ji,t+s}\right)\right]},$$

where $Q_{jt,t+s} = \beta^s P_{jt}C_{jt}/(P_{j,t+s}C_{j,t+s})$ is the stochastic discount factor between period t and $t+s$. And the price indices are now expressed in recursive form:

$$(P_{jjt})^{1-\varepsilon} = \theta(P_{jj,t-1})^{1-\varepsilon} + (1-\theta)(\tilde{P}_{jjt})^{1-\varepsilon},$$
$$(P_{j-jt}^G)^{1-\varepsilon} = \theta(P_{j-j,t-1}^G)^{1-\varepsilon} + (1-\theta)(\tilde{P}_{j-jt}^G)^{1-\varepsilon}.$$

The country-specific productivity and monetary shocks follow the AR(1) process with correlation η:

$$z_{it} = \eta_z z_{i,t-1} + \epsilon_{it}^z, \qquad \mu_{it} = \eta_\mu \mu_{i,t-1} + \epsilon_{it}^\mu$$

where cross-country disturbance terms $\boldsymbol{\epsilon}_t^z$ and $\boldsymbol{\epsilon}_t^\mu$ follow normal distributions with zero mean. Productivity shocks and monetary shocks are mutually independent.

5.1 Optimal Policies

Global loss function. When the pricing stickiness setup changes from one-period Calvo to staggered pricing, the trade-off faced by monetary policy in the baseline model remains unchanged. We can still decompose the global welfare loss function into three parts: non-tradable goods distortion, domestic tradable goods distortion, and foreign tradable goods distortion, which corresponding directly to equation (3.1).

$$\mathbb{E}_0\left(\sum_{t=0}^{\infty}\beta^t(l_t - \tilde{l}_t)\right) = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\underbrace{\frac{1}{2}v\sum_{i=1}^{N} n_i\left(\frac{\varepsilon}{\tilde{\theta}}\pi_{Nit}^2 + (c_{Nit}-\tilde{c}_{Nit})^2\right)}_{\text{related to }(m_{it}-z_{it})^2} + \underbrace{\frac{1}{2}(1-v)\sum_{i=1}^{N} n_i^2\left(\frac{\varepsilon}{\tilde{\theta}}\pi_{iit}^2 + (c_{iit}-\tilde{c}_{iit})^2\right)}_{\text{related to }(m_{it}-z_{it})^2}\right.$$

$$+\underbrace{\frac{1}{2}(1-v)\sum_{i=1}^{N}\sum_{j\neq i}n_i n_j\left(\frac{\varepsilon}{\tilde{\theta}}(\pi^X_{jit})^2+(c_{jit}-\tilde{c}_{jit})^2\right)}_{\text{related to }(m_{xt}-z_{jt})^2,\ x\in\{j,i,1,b,g\}}\Bigg],$$

where $\pi^X_{jit} = p^X_{jit} - p^X_{ji,t-1}$ is the log inflation term and $\tilde{\theta} = (1-\beta\theta)(1-\theta)/\theta$ measure the degree of price stickiness. In the one-period version of Calvo, price dispersion is proportional to consumption deviation so the loss function simplifies to equation (3.1), whereas in the standard Calvo framework, it enters the loss function through inflation.

To eliminate the distortions in non-tradable and domestic tradable goods, monetary policy m_{it} should target z_{it}, whereas eliminating the distortion in foreign tradable goods depends on the pricing paradigms. Under PCP, LCP, DCP, and GCP, the monetary policies m_{jt}, m_{it}, m_{1t}, and m_{gt} need to target z_{jt} for any country j. The policy trade-offs underlying equations (3.2) to (3.5) in the baseline model remain unchanged.

Country loss function. Similarly, we can decompose country loss into three components, each arising from non-tradable goods, domestic tradable goods, and foreign tradable goods.

$$\mathbb{E}_0\left(\sum_{t=0}^{\infty}\beta^t(l_{it}-\tilde{l}_{it})\right)=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg[\underbrace{\frac{1}{2}v\left(\frac{\varepsilon}{\tilde{\theta}}\pi^2_{Nit}+(c_{Nit}-\tilde{c}_{Nit})^2\right)}_{\text{related to }(m_{it}-z_{it})^2}+\underbrace{\frac{1}{2}(1-v)n_i\left(\frac{\varepsilon}{\tilde{\theta}}\pi^2_{iit}+(c_{iit}-\tilde{c}_{iit})^2\right)}_{\text{related to }(m_{it}-z_{it})^2}$$

$$+\underbrace{\frac{1}{2}(1-v)\sum_{j\neq i}n_j\left(\frac{\varepsilon}{\tilde{\theta}}(\pi^X_{jit})^2+(c_{jit}-\tilde{c}_{jit})^2\right)}_{\text{related to }(m_{xt}-z_{jt})^2,\ x\in\{j,i,1,b,g\}}\Bigg]. \tag{5.1}$$

The above country loss function can be seen as the dynamic environment version of equations (3.17) to (3.20). Compared to the global welfare function, each country focuses solely on its own consumption deviation and corresponding inflation, resulting in different weights.

Table 2: The inflation targeting rule of country i, $i \in \{1, 2, ..., N\}$

Pricing Paradigm	Cooperative Game	Nash game
PCP	$\pi_{Nit} = \pi^P_{ijt} = v\pi_{Nit} + (1-v)\sum_{j=1}^N n_j\pi^P_{ijt} = 0$	
LCP	$v\pi_{Nit} + (1-v)n_i\pi_{iit} + (1-v)\sum_{j\neq i}^N n_j\pi^L_{jit} = 0$	
DCP	a special case of GCP	a special case of GCP
BCP	$v\pi_{Nit} + (1-v)n_i\pi_{iit} = 0$	
GCP	$vn_i\pi_{Nit} + (1-v)n_i^2\pi_{iit}$ $+(1-v)\alpha_i\sum_{j=1}^N(n_j(1-n_j)\pi^G_{j-jt}) = 0$	$v\pi_{Nit} + (1-v)n_i\pi_{iit}$ $+(1-v)\alpha_i\sum_{j\neq i}(n_j\pi^G_{j-jt}) = 0$

Policy implementation. The optimal policy implementation rule in the baseline model, as shown in Table 1, remains unchanged, with the focus shifting from targeting prices to targeting inflation, which demonstrates the robustness of our baseline model. There exists the open economy version of the Divine Coincidence Phillips Curve, similar to Rubbo (2023), where targeting the corresponding inflation can eliminate the trade-off between output gaps and inflation. We summarize the operation of monetary policy under PCP, LCP, DCP, BCP, and GCP under staggered prices as follows:

A detailed proof can be found in the appendix. Table 1 corresponds directly to Table 2, presenting the optimal monetary policy implementation rules under various pricing paradigms. Thus, the key conclusions of the baseline model remain robust in the dynamic setting, evident in several aspects: both global and country welfare follow a similar decomposition, policymakers face analogous trade-offs in implementing monetary policy, and the optimal money supply m_{it}^{opt} is consistent across both dynamic and static models (proved in the appendix). Furthermore, for all pricing paradigms, monetary policy implementation remains stable in the dynamic model, shifting from price targeting to inflation targeting. The effects of global currency design are also analogous, ensuring that Proposition 1 still holds under staggered pricing.

5.2 Calibration

Table 3: Calibrated country-specific parameter values

Country Name	Country Code	n	α	α^*	σ_z^2	$\sigma_{\mu,a}^2$	$\sigma_{\mu,b}^2$
United States	USA	0.2940	0.4338	0.2472	0.0568	0.0003	0.0009
Euro area	EMU	0.1976	0.2931	0.1888	0.1063	0.0005	0.0015
China	CHN	0.1501	0.1228	0.1519	0.1298	0.0049	0.0009
Japan	JPN	0.0738	0.0759	0.0814	0.0915	0.0008	0.0040
United Kingdom	GBR	0.0475	0.0744	0.0539	0.1893	0.0005	0.0023
India	IND	0.0298	0	0.0345	0.1831	0.0038	0.0014
Brazil	BRA	0.0282	0	0.0326	0.1722	0.0030	0.0102
Canada	CAN	0.0250	0	0.0291	0.0855	0.0016	0.0009
Korea, Rep.	KOR	0.0224	0	0.0261	0.0520	0.0010	0.0032
Russian Federation	RUS	0.0217	0	0.0253	0.3546	0.0297	0.0114
Australia	AUS	0.0211	0	0.0246	0.0166	0.0026	0.0024
Mexico	MEX	0.0192	0	0.0224	0.1975	0.0012	0.0031
Indonesia	IDN	0.0125	0	0.0147	0.0536	0.0167	0.0023
Turkiye	TUR	0.0124	0	0.0146	0.2699	0.0128	0.0061
Saudi Arabia	SAU	0.0097	0	0.0115	0.2587	0.0756	0.0009
Argentina	ARG	0.0091	0	0.0107	0.5982	0.0862	0.0361
Sweden	SWE	0.0080	0	0.0095	0.1124	0.0004	0.0024
Poland	POL	0.0074	0	0.0088	0.0812	0.0013	0.0041
South Africa	ZAF	0.0054	0	0.0064	0.1513	0.0009	0.0076
Denmark	DNK	0.0051	0	0.0060	0.0735	0.0008	0.0014

Note: Table 3 summarizes country-specific parameters for the 20 economies in our analysis. Column 3 reports the economic size measure n_i, Column 4 presents the the global currency basket shareα_i used in our welfare analysis, and Column 5 displays the theoretically optimal SDR share α_i^* derived from Proposition 1. The optimal weights α_i^* serve solely to demonstrate Proposition 1 but are not used for our calibration.

Our analysis has been based on a potential global currency basket that could be used as an invoicing currency for trade pricing, and we have compared this to the existing DCP that is widely applicable in the current international system. It is not clear exactly how to provide a quantitative comparison of the two systems using contemporaneous international data. In this section we offer a partial analysis by comparing DCP and the other international invoicing practices to an alternative that used the IMF's Special Drawing Rights (SDR) as a potential basket currency to be used for GCP.

Building on the dynamic analysis in Section 5.1, we calibrate the model for 20 representative economies to evaluate welfare implications of using the SDR as the invoicing currency in trade. Parameters are categorized as either global (drawn from existing literature) or country-specific (calibrated using national data). Key parameterizations include: home bias $v = 0.8$ to reflect consumer preference for domestic goods; substitution elasticity $\varepsilon = 8$, implying a pre-subsidy price markup of approximately 15%; discount factor $\beta = 0.995$; price stickiness $\theta = 0.75$; and autocorrelation coefficients $\eta_z = 0.8$ for productivity shocks and $\eta_\mu = 0.75$ for monetary shocks.

Table 3 reports country-specific parameters calibrated using annual data from 2003 to 2020. The country size parameter n_i, which simultaneously denotes the mass of consumers and firms in economy i, is measured by its averaged GDP share relative to the global economy. The global currency basket weight α_i corresponds to the IMF's 2022-2027 SDR valuation cycle. [7] We calibrate cross-country covariance matrices Σ_z and Σ_μ for productivity and monetary shocks. Since $Y_{it} = z_{it}L_{it}$, Σ_z is calibrated using the covariance of per capita GDP data across countries. Monetary shocks μ_{it} in our complete markets framework, where exchange rates satisfy $e_{ijt} = m_{it} - m_{jt}$, capture both price shocks from unexpected money supply and financial shocks from noisy cross-border asset transactions. Thus, we adopt GDP deflator covariance matrix($\Sigma_{\mu,a}$) as our baseline specification for calibrating Σ_μ, while reporting exchange rate covariance matrix ($\Sigma_{\mu,b}$) to ensure robustness. The last three columns of Table 3 report each country's productivity shock variance along with monetary shock variances calibrated using two methods.

5.3 Welfare Analysis

The welfare loss computation method follows Gali and Monacelli (2005) and Rubbo (2023). Economies experience period-0 productivity and monetary shocks, then we compute equilibrium impulse response functions as in Schmitt-Grohé and Uribe (2004) and record each country's welfare losses using equation (5.1). Final welfare loss outcomes are simulation averages over shock realizations preserving Σ_z and Σ_μ.

Figure 8 demonstrates the welfare loss across various countries under four pricing paradigms under cooperative monetary policies. Due to monetary shocks being relatively small compared to productivity shocks, PCP significantly outperforms the other pricing paradigms. LCP performs the worst; in high inflation countries, such as Argentina, it brings huge negative impacts because import consumption is heavily affected

[7] We also show the optimal GCP basket implied by Proposition 1. It is notable that both the US and Euro area would have a much smaller weight than that of the SDR, while China would have a higher weight. Note however that these weight are in general not the optimal shares in presence of both productivity and monetary shocks.

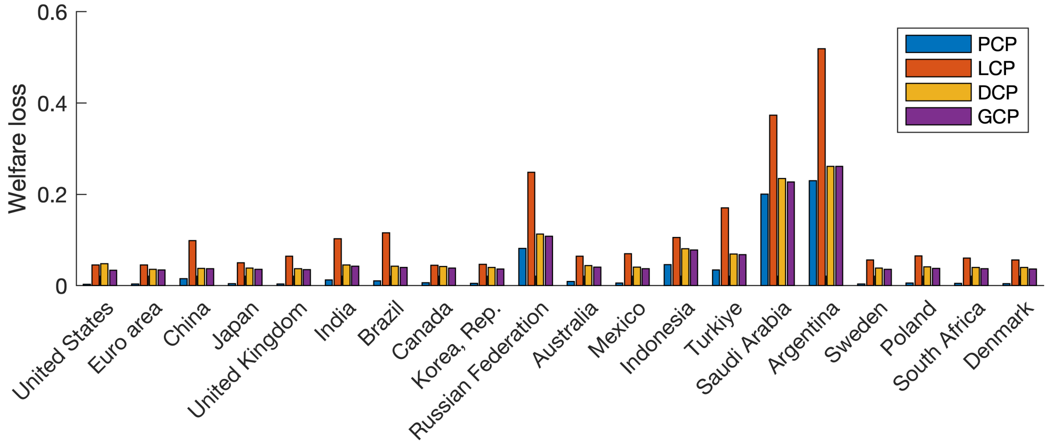

Figure 8: Calibrated countries' welfare loss under cooperation $(\Sigma_z, \Sigma_{\mu,a})$

Note: Figure 8 shows 20 country's welfare loss under cooperative monetary policies. All countries commit to the optimal cooperative monetary policy to deal with productivity shocks while the money supply experience unexpected monetary shocks. The productivity shocks draw from Σ_z *and monetary shocks draw from* $\Sigma_{\mu,a}$*.*

by volatile domestic prices. DCP and GCP, as single-currency pricing specifications, yield welfare levels between those of PCP and LCP.

Table 4 records the percentage of welfare improvement of GCP compared to DCP under two monetary policies. To eliminate the impact of shock differences between countries on welfare, the third column also considers a case where shocks are i.i.d. across countries. It is notable that the US gains far more than any of the other groups. This is because the use of the dollar in international pricing leads the US to skey its monetary policy away from offsetting domestic shocks in an efficient manner. But we note that both the SDR countries and the other non-SDR countries all benefit from a switch to GCP away from GCP, although in these cases the benefits are more modest.

6 Endogenous Currency Choice

In this section, we allow firms the flexibility to choose their preferred currency for export pricing, to examine whether the global currency is supported and to explore the reasons behind its adoption. We are concerned not only with the welfare implications of using the global currency for pricing international trade at the macro level, but also with whether firms will endogenously choose the global currency at the micro level.

6.1 Under Optimal Monetary Policy

The choice of currency invoicing is relevant only for the firms that are unable to change their prices after the realization of shocks. The expected loss for a sticky firm in country j when pricing goods in a currency

Table 4: Relative welfare improvement of GCP compared to DCP

	Cooperative policy			Uncooperative policy		
	$(\Sigma_z, \Sigma_{\mu,a})$	$(\Sigma_z, \Sigma_{\mu,b})$	i.i.d.	$(\Sigma_z, \Sigma_{\mu,a})$	$(\Sigma_z, \Sigma_{\mu,b})$	i.i.d.
United States	29.62%	29.23%	46.91%	23.98%	23.64%	40.82%
Euro area	3.61%	4.17%	7.34%	11.35%	11.31%	17.90%
China	2.92%	4.72%	15.16%	8.24%	11.57%	25.23%
Japan	8.14%	6.45%	15.93%	15.95%	13.06%	26.65%
United Kingdom	6.09%	6.62%	15.74%	14.98%	14.48%	27.28%
India	6.43%	8.29%	17.90%	12.66%	15.17%	28.03%
Brazil	6.47%	5.70%	17.55%	13.01%	10.34%	27.75%
Canada	7.58%	8.88%	18.13%	14.42%	15.81%	28.21%
Korea, Rep.	7.98%	8.02%	17.95%	15.02%	14.16%	28.09%
Russian Federation	4.10%	5.71%	17.64%	6.81%	10.23%	27.84%
Australia	7.93%	8.21%	17.61%	14.49%	14.76%	27.78%
Mexico	8.00%	7.33%	17.40%	14.96%	13.62%	27.61%
Indonesia	3.11%	7.85%	17.92%	7.03%	14.39%	28.04%
Turkiye	1.80%	7.62%	17.95%	6.44%	13.15%	28.08%
Saudi Arabia	3.13%	9.07%	17.83%	4.55%	16.18%	27.97%
Argentina	0.18%	3.10%	17.71%	1.46%	5.51%	27.86%
Sweden	8.21%	8.17%	17.61%	15.58%	14.66%	27.79%
Poland	7.78%	6.90%	17.70%	14.76%	12.91%	27.86%
South Africa	7.70%	6.02%	17.91%	14.85%	11.19%	28.04%
Denmark	8.00%	8.73%	17.65%	15.23%	15.54%	27.83%
Dominant country (USA)	29.62%	29.23%	46.91%	23.98%	23.64%	40.82%
Four other SDR countries	4.38%	5.05%	12.28%	11.50%	12.10%	22.85%
Non-SDR countries	4.67%	6.94%	17.77%	9.11%	12.57%	27.93%
All countries	11.91%	13.58%	27.13%	14.01%	15.71%	30.21%

or currency basket k is given by:

$$\text{var}(m_{kt} - z_{jt}).$$

Therefore, firm ω will choose a currency or currency basket k that minimizes the fluctuations in $m_{kt} - z_{jt}$, which depends on how monetary policy m_{jt} interacts with productivity shocks z_{it}.

6.1.1 Currency choice in the baseline model

In the baseline model with only productivity shocks, we establish a strong presumption in favour of PCP as the optimal invoicing currency for all firms in all countries. Assuming that optimal policy follows that $m_{it} = z_{it}$ under PCP, then we can state:

Lemma 1 *If the money supply follows the optimal PCP policy as $m_{it} = z_{it}$, firms in all countries will prefer PCP over GCP (for any global currency design) and LCP in equilibrium:*

$$PCP \succeq GCP \text{ and } PCP \succeq LCP.$$

In other words, PCP constitutes a stable or self-consistent equilibrium in which policymakers, expecting firms to opt for PCP in their currency choices, design monetary policies based on this assumption, and firms, in turn, adjust their behavior to meet these expectations.

This lemma demonstrates that PCP is an equilibrium pricing decision but does not imply the uniqueness of PCP as a stable pricing strategy. If policy follows an optimal rule implied by GCP in equation (3.15), it is possible that individual firms will also invoice in global currency. The full description of equilibrium conditions in this case is complex, but in the $N = 2$ case, the Appendix establishes that in the baseline model, even when monetary authorities follow (3.15) and the composition of the global currency follows Proposition 1, the all firms will prefer PCP to any other pricing policy. In this case, in the absence of extraneous costs such as those discussed in Devereux et al. (2007), the only self-consistent pricing outcome is PCP.

The validity of Lemma 1 relies on two stringent conditions: (1) all central banks precisely implement the optimal policy, i.e. $m_{it} = z_{it}$, with no deviations caused by monetary shocks; and (2) firms' decisions must be independent of the actions of their competitors or suppliers, meaning there is no price complementarity. Empirical evidence shows that these two conditions are difficult to satisfy, as monetary authorities cannot fully target PPI, and firms' decisions are often shaped by the choices of other firms (Amiti et al., 2014). This creates incentives for firms to adopt third-party currencies, such as the dollar or a global currency, as invoicing currencies. We discuss the case of monetary policy volatility and price complementarity below.

6.1.2 Monetary Shocks

When we allow for monetary velocity shocks as described in Section 4, the endogenous currency choice of firms becomes quite different. To ease exposition, we assume $\sigma_{iz}^2 \equiv \sigma_z^2$ and $\sigma_{i\mu}^2 \equiv \sigma_\mu^2$ primarily to rule out the possibility that the global currency basket includes several highly volatile currencies. For firm-level currency choices, introducing unresponsive velocity shocks will always make GCP more attractive compared to PCP, LCP, and DCP. We present the following lemma:

Lemma 2 *High monetary volatility attracts firms to choose GCP, manifesting in two key aspects:*

(1) For any given monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$, any specific global currency design $\boldsymbol{\alpha}$, and any non-negative value b_r, if firm ω chooses GCP instead of PCP, LCP, or DCP as the invoicing currency when $\sigma_\mu^2/\sigma_z^2 = b_r$, then for any $\sigma_\mu^2/\sigma_z^2 > b_r$, the firm will also choose GCP.

(2) As $\sigma_u^2/\sigma_z^2 \to \infty$, as long as $\alpha_i \neq 1$ for any i, indicating that the global currency is not composed of a single currency, firms will definitely choose GCP instead of PCP, LCP, or DCP, regardless of the monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$.

This lemma emphasizes that as GCP reduces monetary volatility in term of welfare, it also attracts firms to choose it endogenously. The first point of the Lemma 2 shows that higher monetary volatility σ_μ^2 will definitely expand the feasible set of the global currency basket $\boldsymbol{\alpha}$ for firms to choose GCP, regardless of the monetary policy rule $\boldsymbol{a} = [a_{ij}]_{i,j=1}^N$. And the second point demonstrates that as long as monetary volatility is sufficiently high, all firms will choose GCP. The composite structure of the global currency provides GCP with a significant advantage in enhancing monetary stability, a feature not offered by PCP, LCP, or DCP.

Assuming that policymakers in each country anticipate firms will choose PCP and commit to a monetary policy following $m_{jt} = z_{jt} + \mu_{jt}$, there exists a threshold $\Xi_j(\boldsymbol{\alpha})$ for sticky firms in country j:

$$\Xi_j(\boldsymbol{\alpha}) = \frac{1 - 2\alpha_j + \sum_{i=1}^N \alpha_i^2}{1 - \sum_{i=1}^N \alpha_i^2} > 0.$$

If $\sigma_u^2/\sigma_z^2 < \Xi_j(\boldsymbol{\alpha})$, the firm in country j would choose PCP among four pricing paradigms; otherwise, it would opt for GCP. With the introduction of monetary shocks, PCP is no longer a self-consistent and stable pricing paradigm.

6.1.3 Price Complementarity

Micro-level evidence reveals that firms exhibit strong strategic complementarity in their invoicing currency decisions (Amiti et al., 2022, 2014), meaning they tend to align their currency choices with those of competitors or suppliers. In this extension, we incorporate a Kimball aggregator into our baseline model, fully adopting the framework of Mukhin (2022), to examine whether firms adopt a global currency and the motivation behind such a choice. Compared to the baseline model, the consumption bundle is now defined using the Kimball aggregator:

$$\frac{v}{n_i}\int_0^{n_i} \Upsilon\left(\frac{n_i C_{Nit}(\omega)}{vC_{it}}\right) \mathrm{d}\omega + (1-v)\sum_{j=1}^N \left[\int_0^{n_j} \Upsilon\left(\frac{C_{jit}(\omega)}{(1-v)C_{it}}\right) \mathrm{d}\omega\right] = 1,$$

where $\Upsilon(1) = \Upsilon'(1) = 1$ and $h(x) \equiv \Upsilon'^{-1}(x)$ satisfies $h(1) = 1$, $h'(1) = -\varrho$ and $h''(1) = \varsigma$. Apart from this modification, the equilibrium definition remains unchanged, with the detailed system specifications outlined in the appendix.

Considering price complementarity, the expected loss for sticky firms in country j when choosing currency or currency basket k for exports to country i is given by:

$$\min_{k\in\{j,i,1,g\}} \mathrm{var}(m_{kt} - (1-\xi)z_{jt} + \xi(p_{it} - m_{it})), \tag{6.1}$$

where $\xi = (\varsigma - \varrho^2 - \varrho)/(\varsigma - 2\varrho^2)$ captures the degree of price complementarity, and a higher ξ indicates that firms have a stronger incentive to align their pricing currency with that of other firms. The baseline model can be regarded as a special case where $\xi = 0$ at the first-order level.

We examine the self-consistency of PCP after introducing the Kimball aggregator with detail provided in the appendix. Figure 9 presents a simulation exercise in a three-country model, examining the currency choice of a sticky firm in country 3 exporting to country 2. The analysis assumes that governments anticipate

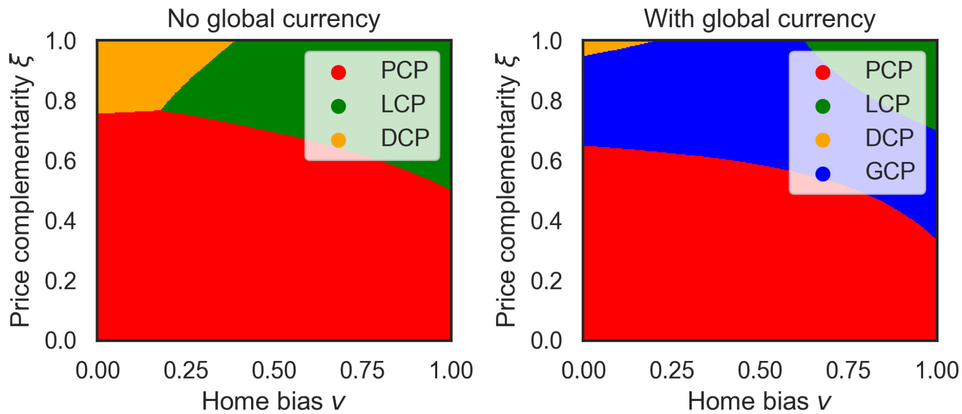

Figure 9: The self-consistency of PCP with price complementarity

Note: Figure 9 illustrates the self-consistency of PCP in a model with three countries ($N = 3$) of sizes $n_1 = 0.5$, $n_2 = 0.3$, and $n_3 = 0.2$, a global currency basket with weights $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, and $\alpha_3 = 0.3$, and parameters $\theta = 0.5$, $\varrho = 1.5$ and $\sigma_{iz} = 1$. It assumes that central banks anticipate all firms adopting PCP and implement the corresponding optimal policy, while firms observe this commitment and expect others to also adopt PCP. The figure examines the currency choice of firms in country 3 exporting to country 2 under these conditions, with the left panel showing choices among PCP, LCP, and DCP, and the right panel including the option of GCP.

firms adopting PCP and commit to the corresponding monetary policy, while the firm expects other firms to also choose PCP, and then its own choice is evaluated under these conditions.[8] The left panel illustrates the case where this firm can choose only among PCP, LCP, and DCP, while the right panel includes the option of GCP. The results show that PCP remains stable only when ξ is relatively low, indicating weak price complementarity among firms.

Therefore, we propose two reasons why PCP might not be a stable equilibrium. First, the composite structure of the global currency helps to stabilize random money supply shock. Second, under price complementarity, firms tend to align their pricing with competitors, leading them to adopt a third-party currency.

6.2 Global Currency Design

Our previous analysis examined whether the optimal monetary policy at the macro level aligns with firms' pricing decisions at the micro level. In this subsection, we assume that each country adopts a monetary policy aimed at stabilizing firms' marginal costs, i.e. $mc_{jt} \equiv 0$ or $m_{jt} \equiv z_{jt}$. We discuss the motivations behind firms' currency choices under this exogenous monetary policy rule and how the global currency basket share impacts these decisions. This part of the analysis fully follows Mukhin (2022), characterized by price complementarity, but we does not include intermediate goods for simplicity.

[8] When price complementarity ξ is considered, firms' choices depend not only on how the monetary policy rule m_{it} responds to z_{it} but also on the choices of other firms. Given the complex interaction between equilibrium and optimal monetary policy, Figure 9 is drawn under the assumption that firms believe others will adopt PCP.

6.2.1 Advantages of four pricing paradigms

Repeating the analysis of Mukhin (2022), we examine the marginal currency choice of a sticky firm in country j exporting to country i under the flexible price limit $\theta \to 0$. In this scenario, the firm's currency (basket) decision, i.e. equation (6.1), simplifies to:

$$\min_{k \in \{j,i,1,g\}} \operatorname{var}(m_{kt} - (1-\xi)z_{jt} - \xi(vz_{it} + (1-v)z_t)). \tag{6.2}$$

Equation (6.2) succinctly explains the reasons behind firms' choices among the four pricing paradigms:

1. **PCP: High covariance between m_{jt} and z_{jt}.** Due to home bias v and the presence of domestic tradable goods, a country's monetary policy m_{jt} tends to respond more strongly to domestic shocks z_{jt}, incentivizing firms in country j to choose currency j. Lower price complementarity ξ increases the covariance between m_{kt} and $(1-\xi)z_{jt}$, thereby favoring PCP.

2. **LCP: High covariance between m_{it} and z_{it}.** Similarly, due to the high correlation between m_{it} and $\xi v z_{it}$, firms in country j are incentivized to choose currency i. Higher price complementarity ξ and greater home bias v can promote LCP.

3. **DCP: Low variance of m_{1t} and high covariance between m_{1t} and z_t.** The 'anchor currency advantage' and 'large economy advantage' of U.S. dollar emphasized by Mukhin (2022) correspond to the low variance of m_{1t} and the high covariance between m_{1t} and z_t, respectively. Since $z_t = \sum_{i=1}^{N} n_i z_{it}$, shocks from large countries dominate the global-weighted productivity shock z_t, giving firms in country j an incentive to choose the currency of a large country. Higher ξ, larger n_1, and lower v increase the correlation between m_{1t} and $\xi(1-v)z_t$, thereby favoring DCP.

4. **GCP: Low variance of m_{gt} and high covariance between m_{gt} and $z_{jt}/z_{it}/z_t$.** As a composite currency $m_{gt} = \sum_{i=1}^{N} \alpha_i m_{it}$, the global currency g features a money supply m_{gt} with lower variance. Depending on the specific design of the global currency, m_{gt} may also exhibit a high correlation with z_{jt}, z_{it}, or z_t. Although the advantages of a global currency depend on its composition, higher price complementarity ξ tends to encourage the adoption of GCP.

6.2.2 From DCP to GCP

We conducted a simulation exercise to examine how the composition of a global currency basket influences its adoption in a three-country model consisting of the US (country 1), the EU (country 2), and the RoW (country 3). At $t = 0^-$, the parameters (v, ξ) are set to produce a stable equilibrium dominated by DCP, where all firms choose the dollar as the pricing currency in international trade, rather than PCP or LCP. At $t = 0$, the global currency with a given basket $(\alpha_1, \alpha_2, \alpha_3)$ becomes available, giving firms the option to price their goods in either the dollar or the newly introduced global currency (with only two choices).

Due to price complementarity, sticky firms' currency choices at $t = 0$ are shaped by their beliefs about which firms would adopt the global currency. A Nash equilibrium (NE) is achieved when firms' beliefs are consistent with their actions, where the actions are determined by the response function in equation (6.1). For a given global currency basket $(\alpha_1, \alpha_2, \alpha_3)$ with potential multiple Nash equilibria, we use an algorithm to identify a feasible one: in the first iteration step ($s = 1$), we guess that all firms will **choose the dollar** and make their pricing decisions accordingly. For $s \geq 2$, we update our guess based on the currency choices of all firms in $s - 1$ and then derive firms' currency choices. The process continues until firms' expectations align with their actions. This algorithm identifies **a NE biased toward low GCP adoption**, as it starts with the initial guess that all firms use the dollar.

We define the function $\mathbf{1}_{\{\mathcal{C}(j,i)\in g\}}$ as equal to 1 if a firm in country j chooses the global currency when exporting to country i, and 0 otherwise. The adoption rate of the global currency is then defined as:

$$\frac{\sum_{j=1}^{N}\sum_{i\neq j}(n_j n_i \mathbf{1}_{\{\mathcal{C}(j,i)\in g\}})}{\sum_{j=1}^{N}\sum_{i\neq j}(n_j n_i)},$$

where $n_j n_i$, the share of trade between countries i and j in the steady state, has been normalized.

The left panel of Figure 10 illustrates the adoption rate of the global currency in a three-country model under various global currency designs. A widely accepted global currency typically has a high dollar weight (high α_1) and a moderate euro weight (moderate α_2). When the weight of non-dollar currencies becomes too high, the adoption rate declines. This is because, under pricing complementarity, firms exhibit inertia or stickiness in their choice of pricing currency and are reluctant to switch. Consequently, a dollar-like global currency is more likely to achieve broad adoption.

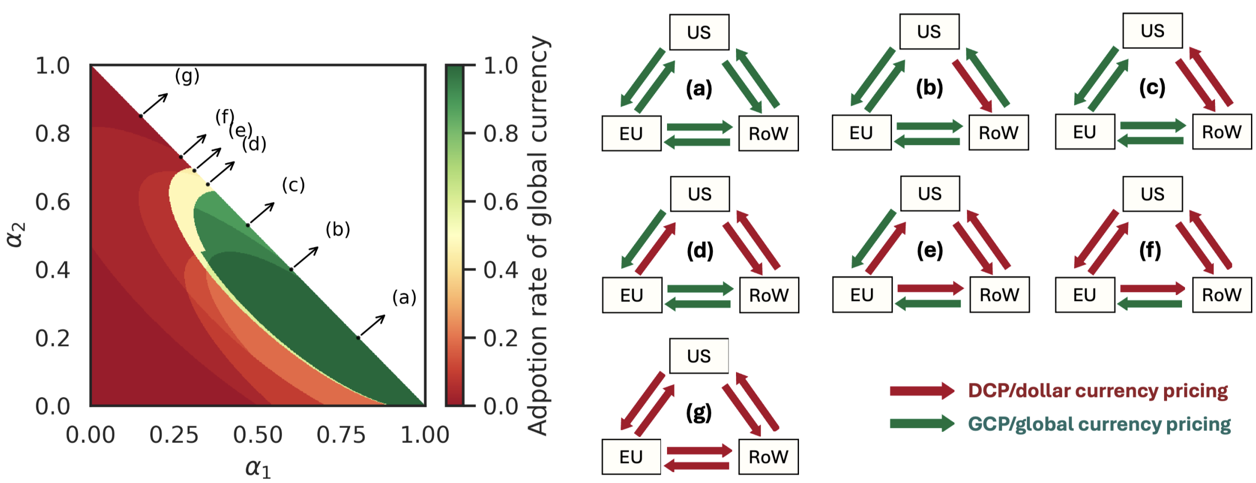

Figure 10: Adoption of the global currency under different designs

Note: Figure 10 illustrates the introduction of a global currency in a three-country model starting from a DCP equilibrium. The parameters are: country sizes $n_1 = 0.6$, $n_2 = 0.35$, and $n_3 = 0.05$; productivity shocks $\sigma^2_{1z} = 1$, $\sigma^2_{2z} = 4$, and $\sigma^2_{3z} = 9$; home bias $v = 0.1$; price complementarity $\xi = 0.8$; monetary policy rule $m_{it} = z_{it}$ and $\theta = 0.2$. The left panel shows the adoption rate of the global currency under various global currency designs, where greener areas represent a higher adoption of the global currency, while redder areas indicate a continued preference for the dollar. The line $\alpha_1 + \alpha_2 = 1$ denotes global currency designs composed solely of the dollar and euro, with the seven labeled points (a) to (g) corresponding to decreasing dollar weight and increasing euro weight. The right panel illustrates the pricing currency choices at each of these seven points, with red arrows indicating trade priced in dollars, while green arrows denoting trade priced in the global currency.

The $\alpha_1 + \alpha_2 = 1$ line in the left panel represents global currency designs composed of only dollar and euro, with points (a) to (g) on the line indicating decreasing dollar and increasing euro shares. The right panel shows trade pricing choices at these points. Point (a) shows that a dollar-like global currency is widely adopted. As the euro's share increases, trade between the U.S. and smaller countries starts using the dollar (points b and c) because the U.S., as a large country, generates stronger pricing complementarity. As the euro's share continues to rise, only transactions destined for Europe adopt the global currency (points e and f), as firms aim to align with local competitors in the eurozone. A euro-like global currency (point g) is not adopted.

6.2.3 The Self-fulfillment of the GCP

Section 6.2.2 focuses on a single NE tilted towards low global currency adoption, driven by firms' pessimistic expectations about its use. However, the introduction of a global currency can lead to multiple equilibria with varying levels of GCP adoption, which we explore in this subsection. Figure 11 identifies two additional Nash equilibria biased towards higher GCP adoption under the same parameters as Figure 10. The key difference in the algorithm lies in the first iteration step ($s = 1$): the left panel initially guesses EU-related trade to adopt the global currency, while the right panel guesses global currency to be adopted across all international trade.[9]

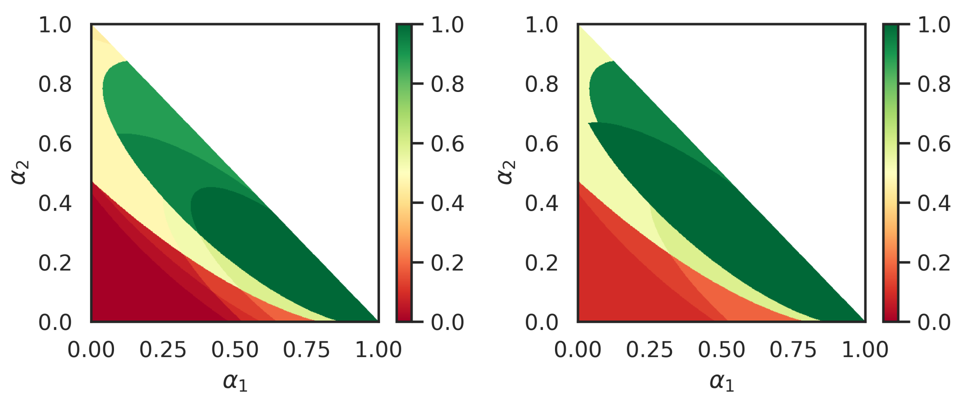

Figure 11: Two other Nash equilibria

Note: Figure 11 identifies two additional Nash equilibria using the same parameters as Figure 10. The NE in Figure 10 can be regarded as firms intially expecting universal dollar use, while the subplots in Figure 11 reflect initial expectations of global currency adoption for EU-related trade (left) and all international trade (right).

Figures 10 and 11 illustrate how belief or sentiments toward global currency adoption shape Nash equilibria. In Figure 10, firms display a pessimistic outlook, resulting in low GCP usage. In contrast, the left and right panels of Figure 11 reflect neutral and optimistic sentiments, respectively, leading to increasingly widespread use of the global currency. Thus, due to the presence of price complementarity, the adoption of the global currency is self-fulfilling — optimistic expectations drive its wider use in international trade. This

[9]Under these parameters, multiple Nash equilibria exist, and we identify only three of them to illustrate that each corresponds to a different GCP adoption rate. The exact number of equilibria in this simulation is not our focus.

underscores the importance of non-economic policies, such as government advocacy and regulatory mandates to encourage GCP usage.

6.3 Intermediate Goods

In the earlier analysis, production relied solely on labor. We now extend the baseline model to incorporate intermediate goods into the production function. Firms in country j uses labor L_j and intermediate goods I_j for production, with intermediate goods having a share of ϕ in the production function:

$$Y_{jt} = \frac{Z_{jt} L_{jt}^{1-\phi} I_{jt}^{\phi}}{(1-\phi)^{1-\phi}\phi^{\phi}},$$

where I_{jt} mirrors the consumption bundle in country j, consisting of the same components as the consumption basket.

Global Currency Pricing. We shows a numerical example of the optimal monetary policy m_{it} in a two-country model, with the monetary policies are given by $m_1 = a_{11}z_1 + a_{12}z_2$ and $m_2 = a_{21}z_1 + a_{22}z_2$. Figure 12 illustrates the response of the optimal monetary policy when $N = 2$, with $n_1 = n_2 = 0.5$, $v = 0$, and $\alpha_1 = 0.7$, $\alpha_2 = 0.3$, as the share of intermediate goods ϕ increases from 0 to 1.

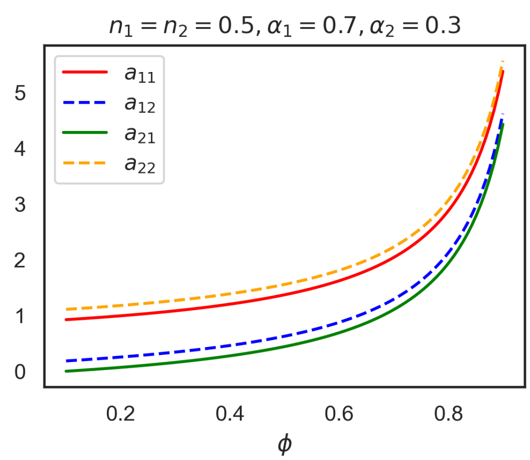

Figure 12: Optimal monetary policy of GCP under cooperative game with production network

Note: The figure 12 shows the response of the optimal monetary policy of when $N = 2, v = 0, n_1 = n_2 = 0.5$ and $\alpha_1 = 0.7, \alpha_2 = 0.3$, as the degree of intermediate goods ϕ changes from 0 to 1.

The introduction of the production network results in two key changes to the optimal monetary policy. First, all currencies react more strongly to productivity shocks (z_{1t}, z_{2t}), as seen by the increase in all lines with rising ϕ in Figure 12. This occurs because monetary policy m_{jt} only partially influences marginal costs, as $mc_{jt} = (1-\phi)m_{jt} + \phi p_{jt} - z_{jt}$, prompting policymakers to adopt more aggressive monetary policies to respond to shocks. Second, each country's monetary policy becomes more responsive to foreign shocks, as indicated by the increase in $a_{12}/(a_{11} + a_{12})$ for country 1 and $a_{21}/(a_{21} + a_{22})$ for country 2 as ϕ increases.

The introduction of the global value chain makes marginal costs dependent on the prices of foreign goods, prompting monetary policy to place greater emphasis on foreign shocks. These two points also hold under all pricing paradigms.

Endogenous currency choice. When using a CES aggregator, even with the introduction of intermediate goods ϕ, PCP remains a stable and self-consistent equilibrium in the absence of monetary shocks. Under PCP, the optimal monetary policy ensures that the marginal cost $mc_j \equiv 0$, leading all firms to adopt PCP as their pricing strategy. This implies that Lemma 1 continues to hold even after incorporating intermediate goods.

When using the Kimball aggregator, the introduction of intermediate goods ϕ alters the comparative advantages of different pricing strategies. The expected loss for a firm in country j exporting to country i under the flexible price limit $\theta \to 0$ is given by:

$$\min_{k\in\{j,i,1,g\}} \operatorname{var}\left(m_{kt} - \frac{1-\xi}{1-\phi v} z_{jt} - \frac{\xi v(1-\phi)}{(1-\phi)(1-\phi v)} z_{it} - \frac{(1-v)(\phi+\xi(1-\phi))}{(1-\phi)(1-\phi v)} z_t\right).$$

Despite the changes in the coefficients of z_{jt}, z_{it}, and z_t, the results remain consistent: PCP dominates under low price complementarity ξ, LCP gains an advantage with high ξ and high v, and DCP prevails with high ξ and low v.

7 Conclusion

This paper has explored the positive and normative consequences of a global currency which could replace the US dollar as the predominant invoicing currency for international trade. We derived the implications for exchange rate pass through and real allocations of the use of the global currency and compared this to existing invoicing assumptions used in the literature, such as PCP, LCP, or the more common DCP. Our key result is that in a cooperative outcome there exists a unique composition of an optimal global currency which allows each country to focus purely on domestic objectives. This global currency compositions replicates an outcome where there was a separate currency used for international trade invoicing completely separate from the monetary policies of the member countries. In an optimal global currency basket, the currency of all countries should be included, large countries should be underweighted, and no one country should have a share greater than 50 percent.

We showed that a global currency could welfare dominate any other pricing system in the presence of financial shocks. This is because the optimal currency basket effectively dilutes the effect of individual countries shocks on global allocations. Calibrating the model to 20 countries, we show that there would be welfare gains from switching from a DCP pricing system to the used of the IMF's SDR.

Finally, we showed the conditions under which there would exist a self-consistent equilibrium under GCP, where each country would choose an optimal monetary policy under GCP and individual firms would optimally choose to invoice all exports in the global currency.

References

Emine Boz, Camila Casas, Georgios Georgiadis, Gita Gopinath, Helena Le Mezo, Arnaud Mehl, and Tra Nguyen. Patterns of invoicing currency in global trade: New evidence. *Journal of International Economics*, 136:103604, 2022.

Michael B Devereux and Charles Engel. Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *The Review of Economic Studies*, 70(4):765–783, 2003.

Richard Clarida, Jordi Gali, and Mark Gertler. A simple framework for international monetary policy analysis. *Journal of Monetary Economics*, 49(5):879–904, 2002.

Charles Engel. Currency misalignments and optimal monetary policy: A reexamination. *American Economic Review*, 101(6):2796–2822, 2011.

Gita Gopinath, Oleg Itskhoki, and Roberto Rigobon. Currency choice and exchange rate pass-through. *American Economic Review*, 100(1):304–336, 2010.

Markus K. Brunnermeier, Harold James, and Jean-Pierre Landau. The digitalization of money. *BIS Working Papers No 941*, 2021.

Mark Carney. The growing challenges for monetary policy in the current international monetary and financial system, 2019.

Jordi Galí. The state of new keynesian economics: a partial assessment. *Journal of Economic Perspectives*, 32(3): 87–112, 2018.

Dmitry Mukhin. An equilibrium model of the international price system. *American Economic Review*, 112(2):650–688, 2022.

Ippei Fujiwara and Jiao Wang. Optimal monetary policy in open economies revisited. *Journal of International Economics*, 108:300–314, 2017.

Varadarajan V Chari, Patrick J Kehoe, and Ellen R McGrattan. Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies*, 69(3):533–563, 2002.

Maurice Obstfeld and Kenneth Rogoff. Global implications of self-oriented national monetary rules. *Quarterly Journal of Economics*, 117(4):503–536, 2002.

Michael B Devereux, Charles Engel, and Peter E Storgaard. Endogenous exchange rate pass-through when nominal prices are set in advance. *Journal of International Economics*, 63(2):263–291, 2004.

Philippe Bacchetta and Eric van Wincoop. A theory of the currency denomination of international trade. *Journal of International Economics*, 67(2):295–319, 2005.

José De Gregorio, Pablo García, Emiliano Luttini, and Marco Rojas. From dominant to producer currency pricing: Dynamics of chilean exports. *Journal of International Economics*, 149:103934, 2024.

Linda Goldberg and Cedric Tille. Macroeconomic interdependence and the international role of the dollar. *Journal of Monetary Economics*, 56(7):990–1003, 2009.

Tony Zhang. Monetary policy spillovers through invoicing currencies. *The Journal of Finance*, 77(1):129–161, 2022.

David Cook and Nikhil Patel. Dollar invoicing, global value chains, and the business cycle dynamics of international trade. *Journal of International Economics*, 145:103839, 2023.

Linda S Goldberg and Cédric Tille. Micro, macro, and strategic forces in international trade invoicing: Synthesis and novel patterns. *Journal of International Economics*, 102:173–187, 2016.

Michael B Devereux, Ben Tomlin, and Wei Dong. Exchange rate pass-through, currency of invoicing and market share. Technical report, National Bureau of Economic Research, 2015.

Dennis Novy. Hedge your costs: Exchange rate risk and endogenous currency invoicing. *Warwick Economic Research Paper*, (765), 2006.

Victor Lyonnet, Julien Martin, and Isabelle Mejean. Invoicing currency and financial hedging. *Journal of Money, Credit and Banking*, 54(8):2411–2444, 2022.

Antoine Berthou, Guillaume Horny, and Jean-Stéphane Mésonnier. The real effects of invoicing exports in dollars. *Journal of International Economics*, 135:103569, 2022.

Wanyu Chung. Imported inputs and invoicing currency choice: Theory and evidence from uk transaction data. *Journal of International Economics*, 99:237–250, 2016.

Gita Gopinath and Jeremy C Stein. Banking, trade, and the making of a dominant currency. *The Quarterly Journal of Economics*, 136(2):783–830, 2021.

Mary Amiti, Oleg Itskhoki, and Jozef Konings. Dominant currencies: How firms choose currency invoicing and why it matters. *The Quarterly Journal of Economics*, 137(3):1435–1493, 2022.

Gita Gopinath and Oleg Itskhoki. Dominant currency paradigm: A review. *Handbook of International Economics*, 6:45–90, 2022.

Matteo Maggiori, Brent Neiman, and Jesse Schreger. International currencies and capital allocation. *Journal of Political Economy*, 128(6):2019–2066, 2020.

Zhengyang Jiang, Arvind Krishnamurthy, and Hanno Lustig. Dollar safety and the global financial cycle. *Review of Economic Studies*, 91(5):2878–2915, 2024.

Zhengyang Jiang, Arvind Krishnamurthy, and Hanno Lustig. Foreign safe asset demand and the dollar exchange rate. *The Journal of Finance*, 76(3):1049–1089, 2021.

Juliana Salomao and Liliana Varela. Exchange rate exposure and firm dynamics. *The Review of Economic Studies*, 89(1):481–514, 2022.

Antonio Coppola, Arvind Krishnamurthy, and Chenzi Xu. Liquidity, debt denomination, and currency dominance. Technical report, National Bureau of Economic Research, 2023.

Luigi Bocola and Guido Lorenzoni. Financial crises, dollarization, and lending of last resort in open economies. *American Economic Review*, 110(8):2524–2557, 2020.

Andres Drenik, Rishabh Kirpalani, and Diego J Perez. Currency choice in contracts. *The Review of Economic Studies*, 89(5):2529–2558, 2022.

Ryan Chahrour and Rosen Valchev. Trade finance and the durability of the dollar. *The Review of Economic Studies*, 89(4):1873–1910, 2022.

Charles Engel and Steve Pak Yeung Wu. Liquidity and exchange rates: An empirical investigation. *The Review of Economic Studies*, 90(5):2395–2438, 2023.

Zhiguo He, Arvind Krishnamurthy, and Konstantin Milbradt. A model of safe asset determination. *American Economic Review*, 109(4):1230–1262, 2019.

Tarek A Hassan. Country size, currency unions, and international asset returns. *The Journal of Finance*, 68(6): 2269–2308, 2013.

Matteo Maggiori. Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–3071, 2017.

Tarek A Hassan and Tony Zhang. The economics of currency risk. *Annual Review of Economics*, 13(1):281–307, 2021.

Pierre-Olivier Gourinchas and Helene Rey. Exorbitant privilege and exorbitant duty. 2022.

Jason Choi, Rishabh Kirpalani, and Diego J Perez. Us public debt and safe asset market power. Technical report, Mimeo, 2023.

Olivier Blanchard. Public debt and low interest rates. *American Economic Review*, 109(4):1197–1229, 2019.

Nicolas Caramp and Sanjay R Singh. Bond premium cyclicality and liquidity traps. *Review of Economic Studies*, 90 (6):2822–2879, 2023.

Markus K Brunnermeier, Sebastian Merkel, and Yuliy Sannikov. Safe assets. *Journal of Political Economy*, 132(11): 000–000, 2024.

Silvia Miranda-Agrippino and Hélene Rey. Us monetary policy and the global financial cycle. *The Review of Economic Studies*, 87(6):2754–2776, 2020.

Maurice Obstfeld and Haonan Zhou. The global dollar cycle. *Brookings Papers on Economic Activity*, 2022(2): 361–447, 2022.

Saleem Bahaj and Ricardo Reis. Jumpstarting an international currency. 2020.

Christopher Clayton, Amanda Dos Santos, Matteo Maggiori, and Jesse Schreger. Internationalizing like china. *American Economic Review*, 2024. forthcoming.

Sebastian Horn, Bradley C Parks, Carmen M Reinhart, and Christoph Trebesch. China as an international lender of last resort. Technical report, National Bureau of Economic Research, 2023.

BISpaper. Project dunbar: Connecting economies through cbdc: Project mbridge, 2022.

Jordi Gali and Tommaso Monacelli. Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3):707–734, 2005.

Jordi Gali and Tommaso Monacelli. Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116–132, 2008.

Bianca De Paoli. Monetary policy and welfare in a small open economy. *Journal of International Economics*, 77(1): 11–22, 2009.

Guillermo A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3): 383–398, 1983.

Maurice Obstfeld and Kenneth Rogoff. Exchange rate dynamics redux. *Journal of Political Economy*, 103(3):624–660, 1995.

Maurice Obstfeld and Kenneth S. Rogoff. Risk and exchange rates, 1998.

Richard Clarida, Jordi Gali, and Mark Gertler. Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, 115(1):147–180, 2000.

J Galí. Monetary policy, inflation, and the business cycle, princeton university press, princeton. *New Jersey*, 2008.

Michael Woodford and Carl E Walsh. Interest and prices: Foundations of a theory of monetary policy. *Macroeconomic Dynamics*, 9(3):462–468, 2005.

Oleg Itskhoki and Dmitry Mukhin. Optimal exchange rate policy. Technical report, National Bureau of Economic Research, 2023.

Olivier Blanchard and Jordi Galí. Real wage rigidities and the new keynesian model. *Journal of money, credit and banking*, 39:35–65, 2007.

Michael B Devereux, Kang Shi, and Juanyi Xu. Global monetary policy under a dollar standard. *Journal of International Economics*, 71(1):113–132, 2007.

Elisa Rubbo. Networks, phillips curves, and monetary policy. *Econometrica*, 91(4):1417–1455, 2023.

Stephanie Schmitt-Grohé and Martın Uribe. Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 28(4):755–775, 2004.

Mary Amiti, Oleg Itskhoki, and Jozef Konings. Importers, exporters, and exchange rate disconnect. *American Economic Review*, 104(7):1942–1978, 2014.