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ABSTRACT

The paper examines whether the US evidence in favour of a nonlinearity in the Phillips curve is robust or fragile. To this end, we use both cross city and aggregate time series data. We are particularly concerned with the possibility that the evidence in favour a nonlinear Phillips curve may is fact be driven by improperly controlling for inflation expectations. Our finding suggest that the evidence in support of a nonlinear Phillips curve is very fragile.

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1 Introduction

Prior to the outbreak of COVID-19 in 2020, a large body of work on inflation came to the conclusion that the Phillips curve was likely quite flat. As a result, it was expected that an episode of temporarily tight labour markets should only have a minor effect of inflation. That view has been put into considerable question in light of the inflation experienced from 2021 to 2023 when we saw the simultaneous occurrence of high inflation, high job vacancies and low unemployment. In particular, Benigno and Eggertsson [2023] has provided considerable evidence suggesting that the Phillips may be strongly nonlinear, with inflation accelerating quickly when the vacancy-to-unemployment ratio becomes greater than one. This view suggests that labour market tightness played a substantial role in generating the high inflation observed in late 2021 and into 2022, and that the subsequent reduction in labour market tightness was central to bringing inflation back down. In other words, this view implies that if the supply shocks induced by COVID-19 and COVID-19 policies were not accompanied by tight labour markets, then inflation would have been substantially lower and that strong interest rate increases may not have been necessary to bring inflation back down.

Given the importance of drawing the right lesson from the recent inflation experience, this paper aims to examine whether the evidence in support of a nonlinear Phillips curve is robust or fragile. We would argue that the evidence is fragile if there are reasonable alternative specifications or data choices which would lead to a very different conclusion. The main reason we believe it is important to look at this issue is because a strong prior that the Phillips curve is highly nonlinear – if this prior is inappropriate – could lead to very poor policy decisions in the future. For example, suppose we face a set of supply shocks similar in intensity to that observed during COVID-19, but that the resulting inflation does not arises in conjunction with tight labour markets. A mis-interpretation of recent experience could lead one to conclude that a strong monetary response may not necessary to bring back inflation, as it should most likely return to normal on its own. In contrast, if the recent evidence in favour of a nonlinear Phillips curve were in fact masking a de-anchoring of short run inflation expectations, in a future episode one may under-appreciate the danger of inflation becoming intrenched in the absence of forceful monetary response.

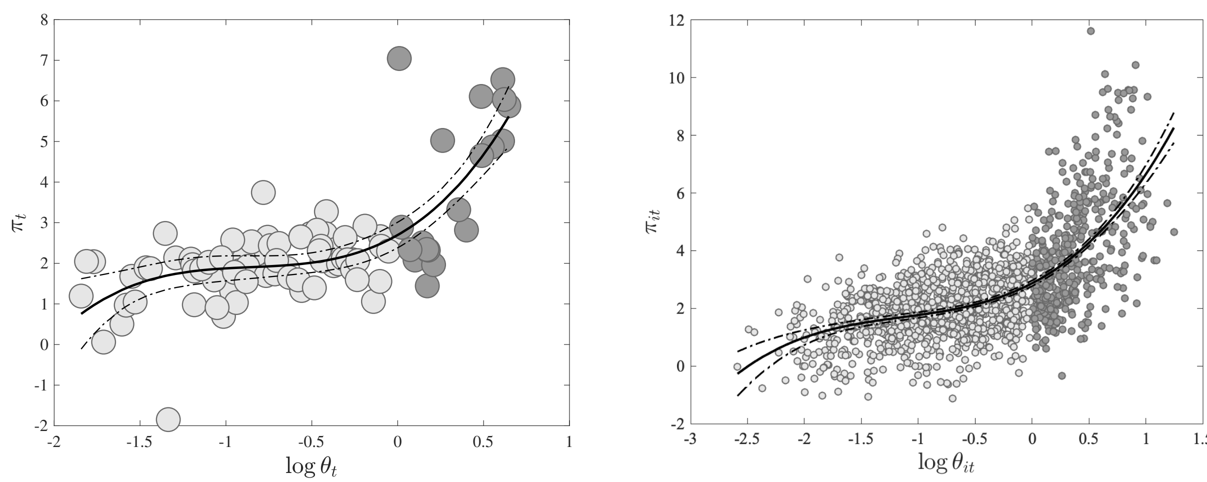
Before we turn to formally exploring different Phillips curve specification, we begin by highlight a set of data patterns which both emphasize the plausibility of a nonlinear Phillips curve as well as its fragility.

A Visual Exploration of Inflation Patterns

The relevant data pattern favouring a rethink regarding the Phillips curve is illustrated in Figure 1, where we plot quarterly observations on year-to-year core CPI inflation against the vacancy to unemployment ratio over the period 2000-2023. Panel (a) presents aggregate observations for the US, while panel (b) presents US city level observations. The city level observations are for 19 major Metropolitan statistical areas (MSA). The details about their construction are presented in Section 2). In both cases, we superimpose an estimated cubic relationship between the two variables to express nonlinearities.

Figure 1: Labour Market Tightness and Inflation, Raw Data

(a) Aggregate data (b) City level data



Notes: Each dot represents a quarter (Panel (a)) or a quarter-city (Panel (b)). Labour market tightness is measured as $\log \theta$, where $\theta = V/U$. Inflation is quarter-to-quarter CPI core inflation (annualized). Light gray dots correspond to $\log \theta < 0$, dark gray dots correspond to $\log \theta > 0$. The black line is the fitted cubic relation between π and $\log \theta$, dotted lines delimit the 95% confidence interval. Sample is 2000Q1-2023Q4 for Panel (a) and 2000Q3-2024Q3 for Panel (b).

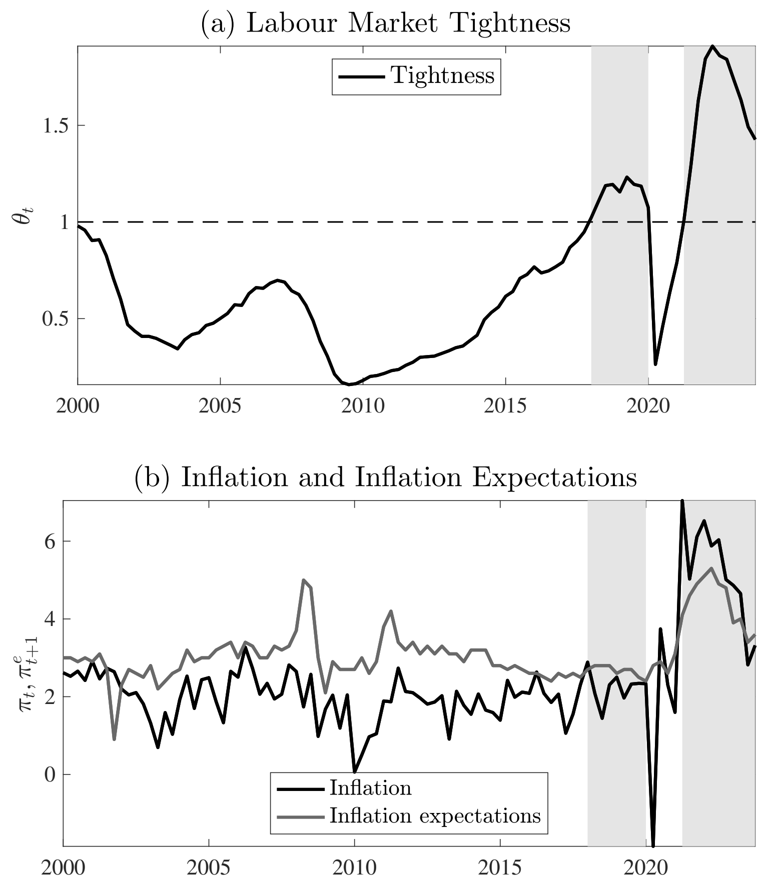
In both Panels (a) and (b) of Figure 1, inflation seems almost unrelated to labour market tightness at low levels of tightness – as would be suggested by a flat Phillips curve view- but a strong positive relationship is apparent at high levels of tightness. In particular when labour market tightness is measured by the vacancy to unemployment ratio, a change in relationship between inflation and tightness appears to arise when the vacancy-to-unemployment ratio is above one. This is clear in both the aggregate data and city level data. In both figures, we have marked in dark data points for which $\theta \geq 1$. Most of these dark dots arise in the post-2021 period (as shown in Figure H.1 in Appendix H). Therefore, the apparent strong positive relationship between inflation and labour market tightness is mostly driven by observations post COVID-19. This intriguing pattern has been interpreted by many, most notably Benigno and Eggertsson [2023] and Gitti [2024], as suggesting a nonlinear Phillips curve, with the effects of labour market tightness on inflation being strong when the vacancy to unemployment ratio rises above one. In this introduction, we present a set of simple figures which illustrate why one should be very hesitant to interpret such observations as providing reliable support for a nonlinear Phillips curve. In the subsequent section, we move beyond simple figures to analyze the question in a more systematic and comprehensive fashion.

When looking at Figure 1, the nonlinear relation between inflation and the vacancy-to-unemployment ratio is very salient, but obviously that does not imply causality. In particular, from a Phillips curve perspective, the high level of inflation post COVID-19 could be due to a combination of factors such as cost shocks, inflation expectations, and labour market tightness. Untangling the respective role of these different forces in driving inflation can be difficult because they tended to move together over the post COVID-19 period. To visualize the potential simultaneity problem, in Figure 2 we plot the time path of the aggregate vacancy-to-unemployment ratio in the upper panel and inflation (actual and expected using the Michigan survey of consumer expectations) in the lower panel. As can be seen, expected inflation tended to be high precisely when the vacancy-to-unemployment ratio was high, which suggests that the high inflation over this period could potentially reflect– at least in part – a short-run de-anchoring of inflation expectations most likely induced by a series of supply shocks.[1]

In order to get a better sense of whether the apparent nonlinearity in the inflation-labour-market tightness relation seen in Figure 1 may be causal, the cross city data has important advantages relative to the time series data. This was the point emphasized in pre COVID-19 study of the Phillips curve by Fitzgerald and Nicolini [2014], McLeay and Tenreyro [2020] and Hazell, Herreño, Nakamura, and Steinsson [2022], and by Gitti [2024] post COVID-19. In particular, the cross sectional data allows one to control for common cost shocks and common aggregate inflation expectations which could be affecting inflation at the same time as a tight labour market. We pursue this in Figure 3, where we plot the residuals of a two-way linear fixed effects regression. In panel (a) we plot city level inflation against city level unemployment-to-vacancy ratio controlling for city fixed effects, while in Panel (b) we control for both city and time fixed effects. Again, we superimpose an estimated cubic relationship between the two variables. As can be seen, the removal of city level fixed effects has very little effect in comparison to Figure 1, and we continue to see a marked nonlinear relationship. In contrast, when removing time fixed effects, as done in Panel (b), we no longer

[1] Beaudry, Hou, and Portier [2024] develop in more depth such an explanation.

Figure 2: Labour Market Tightness, Inflation, Inflation Expectations

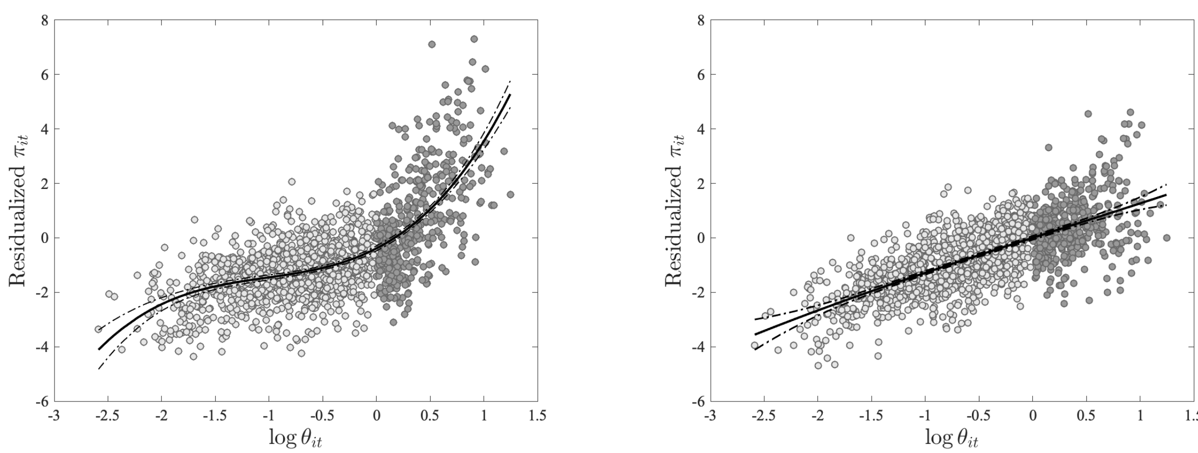


Notes: Labour market tightness is measured as $\log \theta$*, where* $\theta = V/U$*. Inflation is quarter-to-quarter CPI core inflation (annualized). Expectations are one-year-ahead and obtained from the Michigan Survey of Consumers. Grey areas represent quarters with* $\theta \geq 1$*. Sample is 2000Q1-2023Q4.*

see any evidence of nonlinearity. This absence of nonlinearity arises despite the fact that there remains considerable cross-section variation in the local tightness even after removing city and time fixed effects. This pattern indicated that the nonlinear relationship observed in these data in Figure 2 is likely not causal, but instead more likely reflects common factor other than labour market tightness that arrived post COVID-19.

Figure 3: Inflation and Labour Market Tightness, City Level Data

(a) MSA Fixed Effects (b) MSA and Time Fixed Effects



Notes: Each dot represents a quarter-city. Dark dots indicate observations with $\log \theta_{it} \geq 0$ and light dots observations with $\log \theta_{it} < 0$. In Panel (a), residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \kappa \log \theta_{it} + \varepsilon_{it}$ and computed as $\pi_{it} - \alpha_i$. In Panel (b), residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \gamma_t + \kappa \log \theta_{it} + \varepsilon_{it}$ and is computed as $\pi_{it} - \alpha_i - \gamma_t$. The black line is the fitted cubic relation between residualized π and $\log \theta$, dotted lines delimit the 95% confidence interval. Sample is 2000Q3-2024Q3.

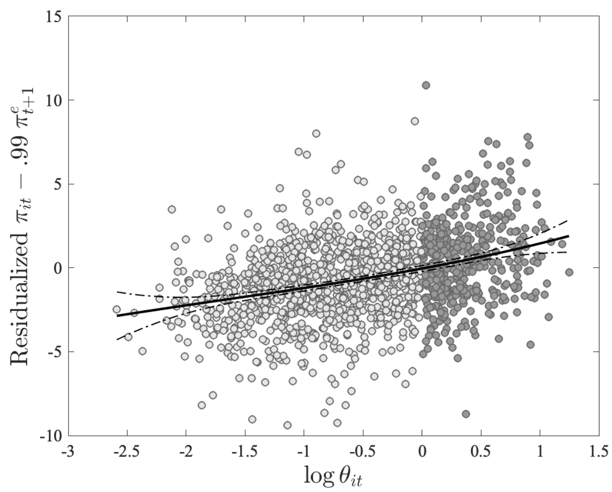
The disappearance of the nonlinearity observed in Panel (b) of Figure 3 relative to Panel (a) could reflect different missing common forces such as costs shocks or movements in inflation expectations. To get a sense for whether inflation expectations may be playing an important role, in Figure 4 we continue to use the cross city data and exploit the theory behind the New Keynesian Phillips curve. In a New Keynesian Phillips curve, inflation expectations should enter the Phillips curve with a coefficient β equal to agents discount factor. This suggest regressing quarter-to-quarter inflation minus β times one-quarter-ahead[2] expected inflation[3] on labour market tightness and city fixed-effects and plotting residualized inflation against $\log \theta$. Using a discount rate of .99, this is what is done in Figure 4.

[2]See Appendix D for how we extract quarter-to-quarter inflation expectations from year-to-year ones.

[3]Here we use aggregate inflation expectations from the Michigan Survey of Consumers. In Section 2, we make use of MSA level inflation expectations and find similar results.

Controlling for inflation expectations this way we again find no sign of nonlinearities.

Figure 4: Inflation and Labour Market Tightness with Expectations, City Level Data



Notes: Each dot represents a quarter-city. Dark dots indicate observations with $\log \theta_{it} \geq 0$ *and light dots observations with* $\log \theta_{it} < 0$. *Residualized inflation is obtained from the two-way linear fixed effects regression* $\pi_{it} - .99\ \pi^e_{t+1} = \alpha_i + \kappa \log \theta_{it} + \varepsilon_{it}$ *and computed as* $\pi_{it} - .99\pi^e_{t+1} - \alpha_i$. *The measure of* π^e_{t+1} *is the national Michigan Survey of Consumers one year-ahead inflation expectation (adjusted to obtain one quarter-ahead expectation (see Appendix D)). The black line is the fitted cubic relation between residualized* π *and* $\log \theta$, *dotted lines delimit the 95% confidence interval. Sample is 2000Q3-2024Q3.*

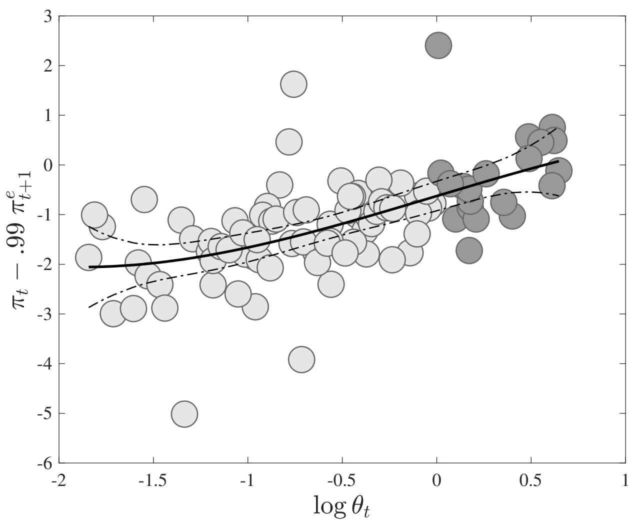
Figures 3 and 4 suggest that the cross-city data provide very little evidence in support of a nonlinear Phillips curve and hint to the possibility that the initially apparent nonlinearity may reflect the confounding effect of increased inflation expectations. The specification of an "Expectation Augmented" Phillips curve was first highlighted by Phelps [1967] and Friedman [1968], and was a key lesson learnt during the inflation episode on the 1970. In parallel to Figure 4, we can do a similar exercise with the aggregate data. This is presented in Figure 5, where we plot inflation minus .99 times expected inflation against the vacancy to unemployment ratio. The nonlinearity –which was quite striking in the raw data in Figure 1– disappears anew.

In Figure 5, we started our sample in 2000, a period where long run inflation expectations are considered reasonably anchored.[4] When long run expectations are anchored, the theory of the New Keynesian Phillips still suggests that inflation expectations remain important for inflation dynamics but, in this case, it is short run inflation expectations that should matter. Accordingly, in Figure 6 we controlled for inflation expectations using short term

[4] Figure H.2 in Appendix H shows similar results for the whole post Volcker period.

measures. When considering a longer sample which includes a period where long run inflation expectations may not have been well anchored, it could be preferable to control for both short and long run expectations. Despite this caveat, in Figure 6 we parallel Figure 5 but using a sample that starts in 1960. Panel (a) presents the raw data for this longer sample, where we again can see evidence of a potential nonlinearity. In Panel (b) we control for expectations as we did in Panel (b) of Figure 5. As we saw in Figure 5, the evidence in favour of of nonlinear relationship disappears in Figure 6 once we control for short run inflation expectations.[5]

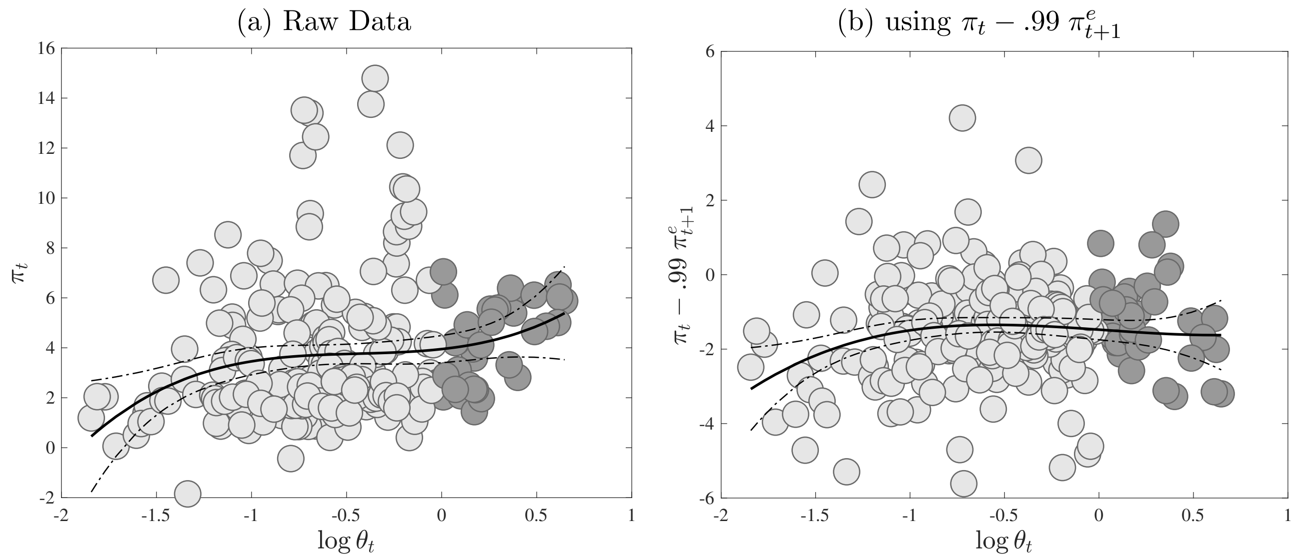
Figure 5: Inflation and Labour Market Tightness with Expectations, Aggregate Data



Notes: Each dot represents a quarter. Dark dots indicate observations with $\log \theta_t \geq 0$ and light dots observations with $\log \theta_t < 0$. Inflation is quarter-to-quarter CPI core inflation (annualized). The measure of π^e_{t+1} is the national Michigan Survey of Consumers one year-ahead inflation expectation (adjusted to obtain one quarter-ahead expectation (see Appendix D)). The black line is the fitted cubic relation between the y-axis variable and $\log \theta$, dotted lines delimit the 95% confidence interval. Sample is 2000Q1-2023Q4.

[5]In Figure H.3 of Appendix H, we show that this holds for various subperiods except 1960-1969, where controlling for expectations does not take away the nonlinearity. As this is a period in which long run expectations were not anchored, the standard New Keynesian Phillips curve is not the right tool for these times.

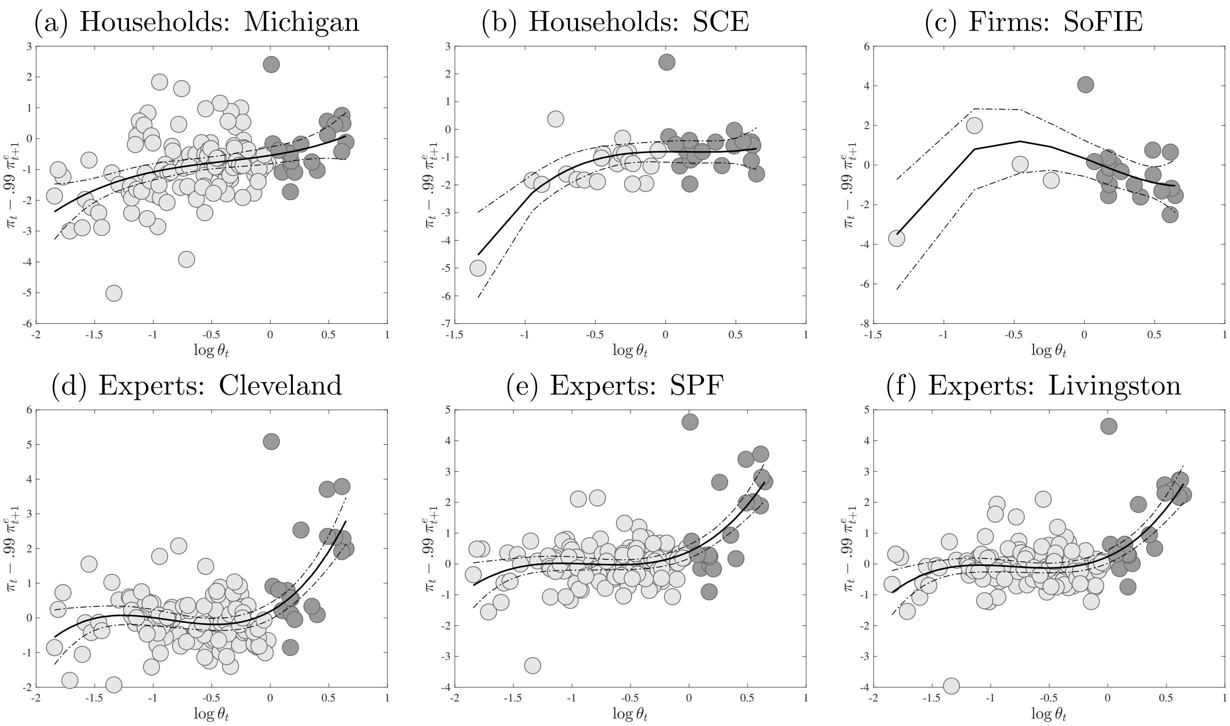
Figure 6: Inflation and Labour Market Tightness, Aggregate Data, Long Sample



Notes: Each dot represents a quarter. Dark dots indicate observations with $\log \theta_t \geq 0$ and light dots observations with $\log \theta_t < 0$. In panel (b), we use Michigan mean inflation expectations as median ones start in 1978. Inflation is quarter-to-quarter CPI core inflation (annualized). The measure of π^e_{t+1} is the national Michigan Survey of Consumers one year-ahead inflation expectation (adjusted to obtain one quarter-ahead expectation (see Appendix D)). The black line is the fitted cubic relation between the y-axis variable and $\log \theta$, dotted lines delimit the 95% confidence interval. Sample is 1960Q1-2023Q4.

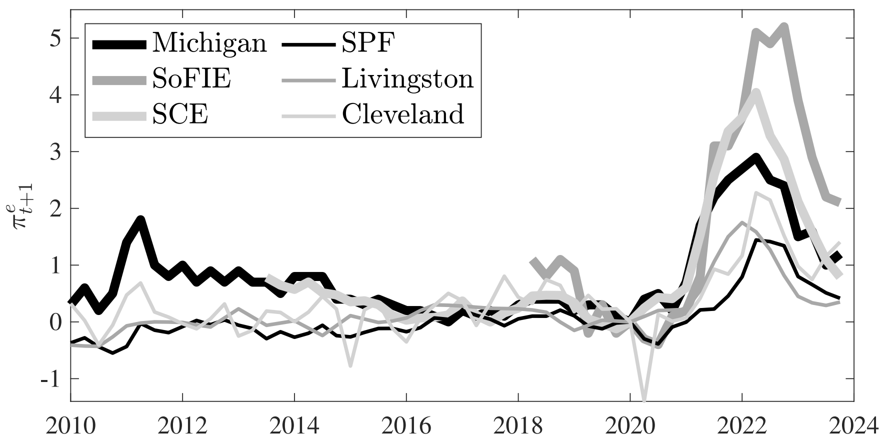
While the cross-city data allowed us to implicitly control for common supply shocks and inflation expectations using time dummies,– without needing to take a stand on the proper measure for inflation expectations– our plots in Figure 5 and 6 based on aggregate data used a specific measure for inflation expectations, that is, one drawn from the Michigan Survey of Consumers and the Survey of Consumers Expectations. Given that measures of inflation expectations can vary across different sources, it is important to verify the correlation patterns in the aggregate inflation data that are robust to controlling for different measure of inflation expectations. This is done in Figure 7 for two types of expectations. We use "experts" expectations (the Cleveland Fed., the Survey of Professional Forecasters and the Livingston survey) on the one side, firms (Survey of Firms Expectations) or households ones (Survey of Consumer Expectations and the Michigan Survey of Consumers) on the other side. In each case, we are controlling for inflation expectations by plotting $\pi_t - .99\ \pi^e_{t+1}$ against log labour market tightness. There is a clear distinction in the results: the nonlinear relationship between inflation and the vacancy-to-unemployment ratio appears robust to controlling for inflation expectations only in the case where we use experts measures. The square and cubic log tightness variables are never significant at 1% for firms and households measures of inflation expectations. The reason for this can easily be understood from Figure 8 where we plot the different measures of inflation expectations over time. In comparison to either household or firm level expectations, experts expectations move much less during COVID-19. Accordingly, controlling for such expectations has little effect relative to the raw data and therefore the nonlinearity in the raw data survives, which explains the results obtained by Benigno and Eggertsson [2023] as they use experts expectations.

Figure 7: Inflation and Labour Market Tightness with Various Measures of Expectations



Notes: Each dot represents a quarter. Dark dots indicate observations with $\log \theta_{it} \geq 0$ *and light dots observations with* $\log \theta_{it} < 0$*. The sample varies with the availability of the measure of expectations we use. Inflation is quarter-to-quarter CPI core inflation (annualized). All inflation expectations are one-year-ahead (adjusted to obtain one quarter-ahead expectation (see Appendix D)). "Cleveland " is the inflation expectations series published by the Federal Reserve Bank of Cleveland, "SPF" is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia, "Livingston" is the Livingston Survey published by the Federal Reserve Bank of Philadelphia, "SCE" is the Survey of Consumer Expectations series published by the Federal Reserve Bank of New York, "SoFIE" is the Survey of Firms' Inflation Expectations series published by the Federal Reserve Bank of Cleveland, "Michigan" is the inflation expectations series of the Surveys of Consumers published by the University of Michigan. Sample always ends in 2023Q4. It starts in 2008Q1 for Michigan, Cleveland and SPF, 2013Q3 for SCE and 2018Q2 for SoFIE. The black line is the fitted cubic relation between the y-axis variable and* $\log \theta$*, dotted lines delimit the 95% confidence interval.*

Figure 8: Various Measures of (mean-adjusted) Inflation Expectations ("Experts" or Firms and Households)



Notes: All inflation expectations are one-year-ahead and are mean-adjusted to take the value 0 on 2020Q1. The thick lines represent firms and households expectations:"MSC" is the Michigan Survey of Consumers published by the University of Michigan, "SCE" is the Survey of Consumer Expectations series published by the Federal Reserve Bank of New York, "SoFIE" is the Survey of Firms' Inflation Expectations series published by the Federal Reserve Bank of Cleveland. The thin lines are "experts" expectations: "Cleveland " is the inflation expectations series published by the Federal Reserve Bank of Cleveland, "SPF" is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia, "Livingston" is the Livingston Survey published by the Federal Reserve Bank of Philadelphia.

Our take away from this visual exploration of data in Figures 1 to 8 is that a nonlinear Phillips curve interpretation of recent inflation outcomes is very fragile. There seems to be little evidence in its support in the cross-section data, and in the time series data controlling for inflation expectations is most often sufficient to explain the apparent nonlinearity. The only case where we do find some evidence in support of a nonlinear Phillips curve interpretation of recent inflation is when focusing on aggregate data and controlling for inflation expectations using expert measures, measures that did a very poor job is predicting inflation over the post 2020 period.[6] In the remaining section, we reexamine all these issues by estimating a large set of Phillips curves specifications using different controls and estimation methods.

2 MSA Level Phillips Curve

In this section, we examine the relationship between CPI inflation and labor market tightness measured by vacancy to unemployment ratio at the MSA level. This exploration of regional Phillips curve relation builds on Hazell, Herreño, Nakamura, and Steinsson [2022]. We first extend the two-sector (tradeable and non-tradeable), multi-location model in Hazell, Herreño, Nakamura, and Steinsson [2022] by incorporating matching friction in the labor market as in Benigno and Eggertsson [2023], so that the labor market tightness is the key measure of labor market condition that affects inflation. A full description of the model is included in Appendix A. This theoretical model gives rise to a regional Phillips curve that takes the following form: [7]

$$\begin{aligned}\pi_{i,t} = &\frac{\beta}{1+\lambda(1-\phi_N)}E_t\pi_{i,t+1} + \frac{\phi_N}{1+\lambda(1-\phi_N)}[\gamma_0\widehat{\theta}_{i,t} + \gamma_1\widehat{\theta}_{i,t}\mathbb{1}_{i,t}] \\ &- \frac{\lambda(1-\phi_N)}{1+\lambda(1-\phi_N)}p_{i,t-1} + \frac{\phi_T}{1+\lambda(1-\phi_N)}X_t + \epsilon_{i,t} \end{aligned} \quad (1)$$

[6] Over the sample 2018Q2-2023Q4 that incorporates the inflation surge and for which all measures of expectations are available, the root-mean-square deviation between expectations and Headline inflation has been lower for firms and households measures (MSC: 1.3, SCE: 2.1. SoFIE: 2.4) that for experts ones (Cleveland: 2.9, Livingston: 2.8, SPF: 2.9).

[7] In closely related work, Gitti [2024] develops and estimates a regional nonlinear Phillips curve with matching friction. Our model differs from Gitti [2024] in that we consider a tradeable and non-tradeable two-sector market structure in each location following Hazell, Herreño, Nakamura, and Steinsson [2022], whereas Gitti [2024] considers a vertical supply chain which leads to a Phillips curve consistent with a one-sector model. More importantly, our work also differs from Gitti [2024] in the empirical strategy in that we include a full set of time-fixed effects as required by our theoretical Phillips curve, whereas Gitti [2024] does not include a full set of time effects, but instead includes year-quarter fixed effects and their interactions with MSA fixed effect using monthly data. We show our baseline results are robust to the use of monthly data in Appendix C.

where $\pi_{i,t} = p_{i,t} - p_{i,t-1}$ and $p_{i,t}$ is the log aggregate local price. $\epsilon_{i,t}$ captures the cost-push shocks and shocks to the matching friction in the labor market. $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ is the coefficient on marginal cost as in the standard New Keynesian model, with $1-\alpha$ being the probability of intermediate firm to adjust prices in a Calvo fairy. $\gamma_0 = \lambda\eta(1-\delta)$ and $\gamma_1 = \lambda\eta\delta$, where η governs the sensitivity of wage to labor market tightness $\widehat{\theta}$, and $\delta \in [0,1]$ captures the rigidity of wage adjustments when the labor market is slack. The estimate on γ_1 then captures whether inflation becomes more or less sensitive to $\widehat{\theta}$ when the labor market becomes excessively tight ($\widehat{\theta} > 1$). ϕ_N and ϕ_T are the expenditure shares of non-tradeable and tradeable goods in the final good basket.

X_t contains the region-invariant variables, including average labor market tightness, aggregate cost-push shocks, and aggregate shocks to the labor market. With panel data, X_t can be captured by a time-fixed effect. Omitting X_t will overestimate γ_0 and γ_1 as (i) the labor market tightness in region i is included in X_t and (ii) if labor market tightnesses across regions are positively correlated, which seems to be true in the MSA-level data.[8]

The detailed derivation of Equation (1) is included in Appendix A.4 and A.5. In our empirical analysis later, we will estimate Equation (1) using MSA-level panel data.

2.1 MSA-level Data

The MSA level CPI series and unemployment numbers are directly available from BLS and JOLTS. However, the number of vacancies is only available at the state level. We follow Gitti [2024] and compute the vacancies in each metropolitan area as the weighted average of vacancies from the states this MSA includes. The weights are the fraction of the population of each state that lives in the corresponding MSA. We compute these weights using U.S. Census 2020. However, our results are robust to using weights computed from 2000 or 2010 Census.

Later in our regression analysis, we also use a shift-share instrument variable following Hazell, Herreño, Nakamura, and Steinsson [2022]. To construct shift-share IV we compute industry-level employment share in each MSA obtained from Census 2000. The industries are at two-digit NAICS level.[9] To compute the employment share, we also need national

[8]Hazell, Herreño, Nakamura, and Steinsson [2022] estimated the Phillips curve of non-tradeable goods prices. As we do not have inflation of non-tradeable goods at MSA-level, we derive the regional Phillips curve of local average inflation that combines both non-tradeable and tradeable goods sectors. As a result, the theoretical coefficient on labor-market-tightness is lower than γ_0 since $\Phi_N < 1$. This is consistent with the findings in Hazell, Herreño, Nakamura, and Steinsson [2022] . In this specification, allowing for a time-fixed effect is crucial. See details in Appendix A.5.

[9]We match the Census 2000 and CPS with industry classification according to 1990 codes, then match the 1990 codes with the two-digit NAICS classification. The 1990 industry codes offer a finer industry classification, using that directly won't change the results qualitatively.

employment levels for each industry. These are obtained from CPS. Due to the availability of unemployment and CPI data at the MSA level, our main sample is 2000 to 2024. Table 1 lists the MSAs we used for our analysis.

Table 1: List of the MSAs in the Sample

Atlanta-Sandy Springs-Roswell	Baltimore-Columbia-Towson
Boston-Cambridge-Newton	Chicago-Naperville-Elgin
Cleveland-Akron	Denver-Aurora-Lakewood
Detroit-Warren-Dearborn	Los Angeles-Long Beach-Anaheim
Minneapolis-St.Paul-Bloomington	New York-Newark-Jersey City
Philadelphia-Camden-Wilmington	Phoenix-Mesa-Scottsdale
Portland-Salem	San Diego-Carlsbad
San Francisco-Oakland-Hayward	Seattle-Tacoma-Bellevue
St. Louis	Tampa-St.Petersburg-Clearwater
Washington-Arlington-Alexandria	

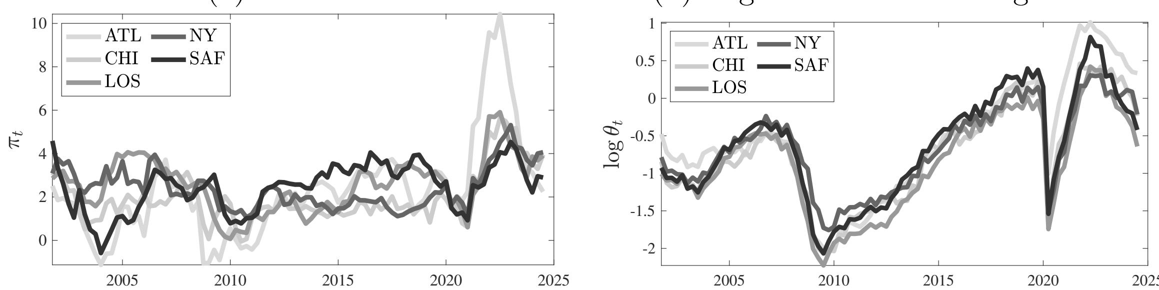
Notes: some of the MSAs like Cincinnatti-Hamilton are not included in the sample as there are no observations of CPI throughout the sample we considered.

The CPI indices for most MSAs are available only bi-monthly.[10] To avoid interpolating the missing data and stay closer to the analysis to Hazell, Herreño, Nakamura, and Steinsson [2022] with quarterly data, we construct quarterly measures of CPI and vacancy-to-unemployment ratios. As implied by our model, we use natural logs of vacancy-to-unemployment ratios as our measure of labor market tightness. In Figure 9, we plot the year-to-year Core CPI and the tightness for five MSAs in our sample. One can see that despite comoving across time, these measures still have substantial variations at the local level.

[10]Only those for Chicago, Los Angeles, and New York are collected monthly.

Figure 9: Core CPI and log tightness for Five MSAs

(a) Inflation (b) Log Labour Market Tightness



Notes: π is year-to-year Core CPI inflation and $\log \theta$ is the natural logs of vacancy-to-unemployment ratios. The five MSAs are Atlanta-Sandy Springs-Roswell, Chicago-Naperville-Elgin, Los Angeles-Long Beach-Anaheim, New York-Newark-Jersey City, and San Francisco-Oakland-Hayward. All series are at quarterly frequency. Sample is 2000Q3-2024Q3.

2.2 Regression Analysis

The specification we estimate follows from our regional Phillips curve (1). It leads to a panel regression with quarterly data:

$$\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log \theta_{i,t} + \beta_{D\times\theta} D_{i,t} \times \log \theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + X_t + \varepsilon_{i,t} \qquad (2)$$

where $\theta_{i,t}$ is the vacancy-to-unemployment ratio for MSA i at time t and $D_{i,t} = \mathbb{1}(\theta_{i,t} \geq 1)$. As suggested in our theoretical regional PC, $p_{i,t-1}$ is the log of price level from one quarter before. From Equation (1), we see that the time fixed effect X_t is needed otherwise the slopes β_θ and $\beta_{D\times\theta}$ would be over-estimated. The MSA fixed effect α_i is not required from the theoretical framework. However, we include it to absorb any time-invariant but MSA-specific factors. In Equation (2), the $\pi^e_{i,t+1}$ is the one-quarter-ahead inflation expectation for each MSA i. We can either follow Hazell, Herreño, Nakamura, and Steinsson [2022] to iterate Equation (1) forward under Rational Expectation and impose a common long-run expectation across MSAs so that its impact will be absorbed by X_t, or we can use some measure of short-run expectation at MSA level. In our baseline results, we estimate an iterated forward version of Equation (2). The results are shown in Table 2.

In Table 2, we see that the estimates on the interaction term, $\beta_{D\times\theta}$, are sizable and significant when we only control for MSA fixed effects but omit time fixed effects, indicating the presence of nonlinearity in the slope of Phillips curve. This is robust to using different measures of CPIs. However, the estimates become small and insignificant once we include time-fixed effects. This pattern is consistent with our simple graphical illustration presented

Table 2: Estimation of Iterated Forward (2) with OLS

	Quarter-to-quarter				Year-to-year			
	Core CPI		HL CPI		Core CPI		HL CPI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_θ	1.19***	1.61***	1.13***	1.03***	0.89***	1.24***	1.08***	0.86***
	(0.145)	(0.328)	(0.209)	(0.328)	(0.071)	(0.154)	(0.092)	(0.137)
$\beta_{D\times\theta}$	2.86***	-0.43	3.71***	-0.15	3.81***	0.03	5.11***	0.08
	(0.457)	(0.586)	(0.665)	(0.586)	(0.220)	(0.270)	(0.288)	(0.241)
β_D	-0.18	-0.21	0.06	-0.16	-0.27**	-0.23**	-0.24	-0.23**
	(0.241)	(0.252)	(0.349)	(0.253)	(0.118)	(0.118)	(0.154)	(0.106)
$p_{i,t-1}$	-0.00	-0.01***	-0.01***	-0.02***				
	(0.001)	(0.005)	(0.002)	(0.006)				
$p_{i,t-4}$					-0.00*	-0.04***	-0.03***	-0.05***
					(0.003)	(0.010)	(0.003)	(0.010)
Observations	1555	1555	1555	1555	1492	1492	1492	1492
MSA F.E.	Y	Y	Y	Y	Y	Y	Y	Y
Time F.E.	N	Y	N	Y	N	Y	N	Y

Notes: We estimate $\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log\theta_{i,t} + \beta_{D\times\theta} D_{i,t} \times \log\theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + X_t + \varepsilon_{i,t}$ *by iterating expected inflation forward and assume common across MSAs long-run expectation anchored by monetary policy as in Hazell, Herreño, Nakamura, and Steinsson [2022]. Columns (1) to (4) use quarter-to-quarter CPI, Columns (5) to (8) use year-to-year CPI at quarterly frequency. To avoid endogeneity issue we include* $p_{i,t-4}$ *instead of* $p_{i,t-1}$ *when using year-to-year inflation. Sample is 2000Q3-2024Q3.*

in the previous sections. The estimates on past price levels are estimated to be negative, as suggested by our theoretical regional Philips curve. Moreover, the small estimates on these past price levels imply a very small λ, indicating a quite flat slope on the traditional Phillips curve.

From our theoretical regional Phillips curve, the error term in regression (1), $\epsilon_{i,t}$ contains cost-push shocks ($-z_{i,t}^K$) and shocks to matching friction at the local labor market ($\widehat{\nu}_{i,t}$). Although aggregate shocks are absorbed by the time-fixed effects, these shocks may be correlated to local market tightness, thus creating an endogeneity problem. We follow Hazell, Herreño, Nakamura, and Steinsson [2022] to use a shift-share IV and redo our previous estimations. In each of the fixed effect regressions, we instrument the tightness and the interaction term with the shift-share IV and its product with the dummy variable $D_{i,t}$.[11] Table 3 reports these results.

Table 3: Estimation of Iterated Forward (2) with IV

	Quarter-to-quarter				Year-to-year			
	Core CPI		HL CPI		Core CPI		HL CPI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_θ	1.07***	2.42*	0.43	1.31	1.04***	2.33***	0.59	1.94***
	(0.361)	(1.293)	(0.541)	(1.251)	(0.132)	(0.646)	(0.385)	(0.549)
$\beta_{D\times\theta}$	25.94***	-7.64*	39.63***	-6.24	9.98***	-2.16	33.45***	-2.18
	(7.136)	(4.582)	(10.625)	(4.536)	(2.781)	(2.029)	(7.770)	(1.831)
β_D	-6.55***	0.56	-9.20***	0.55	-2.28***	-0.19	-8.23***	-0.17
	(1.953)	(0.712)	(2.901)	(0.719)	(0.801)	(0.323)	(2.265)	(0.295)
$p_{i,t-1}$	-0.02***	-0.01	-0.04***	-0.02**				
	(0.006)	(0.008)	(0.009)	(0.008)				
$p_{i,t-4}$					-0.02**	-0.03	-0.10***	-0.03*
					(0.007)	(0.016)	(0.019)	(0.015)
Observations	1440	1440	1440	1440	1422	1422	1422	1422
MSA F.E.	Y	Y	Y	Y	Y	Y	Y	Y
Time F.E.	N	Y	N	Y	N	Y	N	Y

Notes: We estimate $\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log\theta_{i,t} + \beta_{D\times\theta} D_{i,t} \times \log\theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + X_t + \varepsilon_{i,t}$ *by iterating expected inflation forward and assume common across MSAs long-run expectation anchored by monetary policy as in Hazell, Herreño, Nakamura, and Steinsson [2022]. Both* $\log\theta_{i,t}$ *and* $D_{i,t}\times\log\theta_{i,t}$ *are instrumented with shift-share IV and its product with* $D_{i,t}$*. Columns (1) to (4) use quarter-to-quarter CPI, Columns (5) to (8) use year-to-year CPI at quarterly frequency. To avoid endogeneity issue we include* $p_{i,t-4}$ *instead of* $p_{i,t-1}$ *when using year-to-year inflation. Sample is 2000Q3-2024Q3.*

[11] We report the first-stage F-statistics of the instrumented variables on our instruments in Appendix B. Our F-statistics are higher than 100, which indicates a strong set of instruments.

From Table 3, we see that using IV, the estimates on both tightness and the interaction terms are higher than those from OLS. This is likely due to OLS omitting supply shocks, which will induce negative bias in the estimates of tightness. However, like in the results with OLS, the interaction term is only significant when we omit the time-fixed effects. When we include both MSA and time-fixed effects, there is no sign of nonlinearity in the slope of the regional Phillips curve.

As suggested in Hazell, Herreño, Nakamura, and Steinsson [2022], one reason why including time-fixed effects in our previous regressions is essential is to control for the impact of inflation expectations. We then construct MSA-level inflation expectation measures using the micro-level data in the Survey of Consumer Expectations (SCE) from the Federal Reserve of New York. The SCE contains commuting-zone information for each survey respondent. We match each commuting zone to the MSA using the county it belongs to.[12] Then we compute the MSA level average inflation expectation using the median of one-year-ahead inflation expectations for the individuals who belong to the corresponding MSA. Our sample starts from 2013Q3, as this is the earliest date SCE is available. With the MSA level expectation measure, we directly estimate Equation (2) using quarter-to-quarter Core CPI. These results are reported in Table 4.

In Table 4, we start by showing our baseline results in this shorter sample with or without time fixed effects in Columns (1) and (2). The results are consistent with our baseline: the non-liearity disappears once time fixed effects are included. In Column (3) we add MSA level expectations as regressor and omitting time fixed effects. We see that the estimates are in-line with Column (2) and the estimate on the interaction term is low and not significantly different from zero, even without time fixed effects. Finally, as $\pi^e_{i,t+1}$ might be biased and we lack of instrumental variables for it, in Columns (4)-(6) we move $\beta_{\pi^e}\pi^e_{i,t+1}$ to the other side of the equation and assume $\beta_{\pi^e} = 0.99$. We then regress the inflation net of expectation on tightness, interaction terms and price control. In Column (4) we use OLS with only MSA fixed effect, in Column (5) we include both MSA and time fixed effects, and in Column (6) we instrument tightness and interaction term with the shift-share IV and its product with $D_{i,t}$. The results suggest that the steepening of regional PC slope disappears when we control for inflation expectations at the local level.

[12]Using this approach we can match all the commuting zones to a unique MSA except for Riverside, which has the same commuting zone code as Los Angeles. As a result, we drop Riverside from the MSA sample.

Table 4: Estimation of Equation (2) with Local Expectations

	Quarter-to-quarter Core CPI					
	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}			0.76***			
			(0.072)			
β_θ	1.86***	1.85***	1.85***	1.85***	2.04***	1.34
	(0.336)	(0.640)	(0.313)	(0.315)	(0.667)	(1.067)
$\beta_{D\times\theta}$	2.09***	-0.68	-0.30	-1.02*	-1.23	3.52
	(0.628)	(0.758)	(0.627)	(0.588)	(0.790)	(4.177)
β_D	-0.44	-0.09	-0.19	-0.11	0.17	-0.70*
	(0.310)	(0.297)	(0.289)	(0.290)	(0.310)	(0.420)
$p_{i,t-1}$	0.00	-0.02	-0.00	-0.00	-0.02*	-0.02
	(0.004)	(0.010)	(0.004)	(0.004)	(0.010)	(0.011)
Observations	741	741	741	741	741	741
MSA F.E.	Y	Y	Y	Y	Y	Y
Time F.E.	N	Y	N	N	Y	N

Notes: We estimate $\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log\theta_{i,t} + \beta_{D\times\theta} D_{i,t} \times \log\theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + \Theta_t + \varepsilon_{i,t}$. *Column (1) and (2) are estimations without* $\pi^e_{i,t+1}$. *Column (3) reports results with MSA level* $\pi^e_{i,t+1}$ *but without time fixed effects. Columns (4)-(6) use* $\pi_t - 0.99 \times \pi^e_{i,t+1}$ *as dependent variable. Column (6) is instrumenting for* $\log\theta$ *and the interaction term with shift-share IV and its product with* $D_{i,t}$*. Sample is 2013Q3-2024Q3.*

3 Aggregate Data

As nonlinearities in the Phillips curve may arise from general equilibrium effects not captured by cross-MSA variations, we now estimate an aggregate New Keynesian Phillips curve and introduce nonlinearities int the same way we did in Equation (1). The Phillips curve is piecewise-linear Phillips curve, where the potential kink occurs when labour market tightness θ becomes larger than one, following Benigno and Eggertsson [2023]:

$$\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta}D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v}D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t \quad (3)$$

where π_t is core quarter-to-quarter inflation, π^e_{t+1} is a measure of next quarter inflation expectation, $\theta_t = \frac{V_t}{U_t}$, v_t is a measure of supply shocks and D_t a dummy variable that takes one if $\theta_t \geq 0$. In Appendix G, we show that results are preserved if we use a more flexible form of nonlinearity, more precisely a cubic relation between inflation and log labour market tightness. We restrict estimation of Equation (3) to the post-Volcker period when long run expectations can reasonably be thought as stable. In Appendix F, we also consider the post-2008 period and a longer sample that starts in 1960. We obtain similar results.

Table 5 shows estimation results for various versions of Equation (3) when we omit some variables. We first use an experts measure of expectations (the Federal Reserve Bank of Cleveland) in Columns (1) to (4), and finds a significant steepening when $\theta > 1$ (coefficient $\beta_{D\times\theta}$). With this experts measure of expectations, Table 5 shows that the significance only weakens when one allows the tight labour market dummy D and past inflation to enter in the equation. When one uses the households MSC measure of expectations (Columns (5) to (8)), the coefficient $\beta_{D\times\theta}$ is never significantly positive, and significantly negative (at 10%) in the extended model (8).

The fragility of the nonlinearity is confirmed when we instrument tightness and lagged inflation by their first and second lag[13], as shown in Table 6. Again,$\beta_{D\times\theta}$ becomes insignificant with the expert measure of expectations with the more comprehensive specification (Column (4)) of Table 6.

[13]Mavroeidis, Plagborg-Møller, and Stock [2014] have discussed the fact that lagged macro instruments are weak instruments, which can lead to large sampling uncertainty and sensitivity of parameter estimates to minor changes in specification choices or in the sample period. For a shorter sample starting in 2008, as we have a consistent series monetary policy shocks (as obtained from Bu, Rogers, and Wu [2021]), we run in Appendix F estimations using these monetary policy shocks as instruments (as suggested by Barnichon and Mesters [2020] and done in Beaudry, Hou, and Portier [2023]), and find similar results.

Table 5: Estimation of Equation (3), OLS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	0.86***	0.84***	0.91***	0.71***	0.98***	1.08***	1.09***	0.71***
	(0.08)	(0.08)	(0.08)	(0.10)	(0.13)	(0.14)	(0.14)	(0.14)
β_θ	0.08	0.12	-0.07	-0.03	0.74***	0.77***	0.74***	0.47***
	(0.17)	(0.17)	(0.18)	(0.17)	(0.19)	(0.18)	(0.19)	(0.17)
$\beta_{\theta\times D}$	4.05***	4.00***	2.38***	1.26	0.01	-0.25	-0.56	-1.84*
	(0.65)	(0.64)	(0.87)	(0.94)	(0.84)	(0.83)	(1.15)	(1.04)
β_v		-0.01	-0.01	-0.01		-0.05***	-0.05***	-0.04**
		(0.01)	(0.01)	(0.01)		(0.02)	(0.02)	(0.02)
$\beta_{v\times D}$		0.14***	0.17***	0.14***		0.10*	0.10*	0.08
		(0.05)	(0.05)	(0.05)		(0.06)	(0.06)	(0.05)
β_D			0.99***	0.96***			0.17	0.50
			(0.37)	(0.36)			(0.42)	(0.37)
$\beta_{\pi_{-1}}$				0.23***				0.45***
				(0.08)				(0.07)
N	144	144	144	144	144	144	144	144
adj. R^2	0.60	0.62	0.64	0.65	0.48	0.51	0.50	0.61

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$. *Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for* $\log\theta_t > 0$ *and v is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 1988Q1-2023Q4.*

Table 6: Estimation of Equation (3), IV

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	0.88***	0.83***	0.89***	0.35	0.86***	0.92***	0.92***	0.23**
	(0.11)	(0.11)	(0.12)	(0.25)	(0.26)	(0.26)	(0.26)	(0.11)
β_θ	-0.08	0.03	-0.14	-0.19	0.66**	0.64**	0.67**	0.07
	(0.25)	(0.25)	(0.31)	(0.23)	(0.30)	(0.26)	(0.29)	(0.15)
$\beta_{\theta\times D}$	4.07***	4.14***	3.02*	0.74	0.50	0.63	0.83	-1.86
	(0.66)	(0.60)	(1.57)	(1.73)	(1.01)	(1.06)	(1.34)	(1.17)
β_D			0.72	0.32			-0.13	0.53
			(0.92)	(0.67)			(0.53)	(0.53)
β_v		-0.01	-0.01	-0.01		-0.05***	-0.05***	-0.01
		(0.01)	(0.01)	(0.01)		(0.02)	(0.02)	(0.01)
$\beta_{v\times D}$		0.15***	0.14**	0.06		0.12***	0.11***	0.06*
		(0.05)	(0.06)	(0.04)		(0.04)	(0.04)	(0.03)
$\beta_{\pi_{-1}}$				0.66***				0.90***
				(0.24)				(0.07)
Observations	144	144	144	144	144	144	144	144

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$. *Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for* $\log\theta_t > 0$ *and v is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* $\log\theta_t$ *and* π_{t-1} *are instrumented by their two first lags. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses.* *All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses.* *Sample is 1988Q1-2023Q4.*

4 Conclusion

The main message of this paper is that there are two very different interpretations of the recent inflation experience and they are hard to disentangle. Being cautious, one should dismiss neither readily until better data or methods can more definitely resolve this issue and, most importantly, one should embrace this ambiguity if one wants to avoid policy errors in the future. On the one hand, there is the view that– in addition to supply shocks– labour market tightness played a very important role in generating high inflation post 2020 because the Phillips curve is highly nonlinear and the labour market was very tight. On the other hand, there is the view that the Phillips is likely quite flat and that a de-anchoring of short run inflation expectations following the supply shocks likely played a central role in realized inflation dynamics. The main difficulty is differentiating between these two view, and weighing their respective merits, relates to the difficulty of knowing how to properly control for inflation expectations. The main reason it is so important to differentiate between these two views is that they could lead to different policy responses when faced with stagflation.

To addressed this issue, we began by looking at cross-city data. One of the advantage of using cross-city data is that common inflation expectations can be controlled by the use of time dummies, without a need to take a firm stand on how to measure inflation expectations. However, from the city level data, we found no evidence in support of a nonlinear Phillips curve once a compete set of time dummies were allowed. This by itself should question the reliability of the aggregate evidence in support of a nonlinear Phillips curve. However, it could be possible that the nonlinearity of the Phillips curve is only an aggregate phenomena for which cross city data is not informative.

When looking in detail at the aggregate level evidence, we showed how the evidence for or against a nonlinear Phillips curve was highly sensitive to which measure of inflation expectations one considers most relevant in determining of inflation.[14] If one has a strong prior that the inflation expectations of professional forecasters are the most relevant for thinking about inflation, one can come to the conclusion that the Phillips is highly nonlinear, labour market tightness was a very important driver of inflation post COVID-19 and that inflation expectations played a minor role. In contrast, if one thinks that firm or household expectations are more relevant for thinking about inflation (and implicitly wage) determination, then one comes to the opposite conclusion: the Phillips curve appears linear, quite flat and that short run inflation expectations played an important role in recent inflation dynamics because they can de-anchor easily following supply shocks. Bottom line: the evidence in favour of a highly nonlinear Phillips curve is fragile and one should remain vigilant of the

[14] Reis [2023] discusses how to best navigate with various measures of inflation expectations.

potential role of short run expectations de-anchoring following supply shocks.

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Appendix

A Model Appendix

A.1 Model Setup

We first present the setup of our two-sector, multi-location, New Keynesian model and derive the implied regional Phillips curve that we later estimate with MSA-level data. There are N different regions indexed by i. In each region i, there is a representative household that consumes a final good basket and provides labor. There are two sectors in the economy at each location – tradeable and non-tradeable goods sectors. The final good is produced with both tradeable and non-tradeable goods. There are local monopolistic competitive firms producing varieties of tradeable and non-tradeable goods. In each region, there is a single labor market. The household decides the labor market participation, and a local employment agency matches the workers who search for jobs with the local tradeable and non-tradeable firms, as in Benigno and Eggertsson [2023].

A.1.1 Household

In each region i, there is a representative household that consumes a final good basket $C_{i,t}$. There is a continuum of household members that have different disutilities of participating in the labor market. The household chooses how many members to participate – the labor force participation rate. The household has a GHH preference as below:[15]

$$u(C_{i,t}, F_{i,t}) = \frac{1}{1-\sigma}\left(C_{i,t} - \chi_{i,t}\int_0^{F_{i,t}} f^{\omega} df\right)^{1-\sigma}$$

where $C_{i,t}$ is final goods consumption, $F_{i,t}$ is the measure of members that the household decides to participate in the labor force. The household member is indexed by f, and ranked in order with their disutilities of entering the labor force. $\chi_{i,t}$ is a shock to labor force participation, and $\omega > 0$ measures disutility of participating, which follows from Galí [2011]. Note that

$$\int_0^{F_{i,t}} f^{\omega} df = \frac{F_{i,t}^{1+\omega}}{1+\omega}$$

The labor matching process follows from Benigno and Eggertsson [2023]. At the beginning of the period t, a fraction $(1-s)$ of the labor force is employed, denoted as $N^s_{i,t}$, and the rest

[15] As discussed in Hazell, Herreño, Nakamura, and Steinsson [2022] and Benigno and Eggertsson [2023], using GHH preference shuts down wealth effects on the choice of labor force participation, which simplifies our exposition.

becomes jobless, denoted as $U^s_{i,t}$. They search for work in period t and how many of them match with a job depends on the number of vacancies, $V^s_{i,t}$ posted by the local employment agency, and the matching technology:

$$M_{i,t} = m_{i,t}(U^s_{i,t})^{\mu}(V^s_{i,t})^{1-\mu}$$

where $m_{i,t} > 0$ is matching efficiency and $0 < \mu < 1$. Two special cases arise with this formulation: (1) if $s = 0$ we are back to the standard NK model and (2) all people will need to search for job every period if $s = 1$. Labor market tightness is defined by $\theta_{i,t} \equiv V_{i,t}/U^s_{i,t}$. The job-finding rate is then given by:

$$f(\theta_{i,t}) = \frac{M_{i,t}}{U^s_{i,t}} = m_{i,t}\theta_{i,t}^{1-\mu}$$

After the matching process finished, local employment $N_{i,t}$ and unemployment $U_{i,t}$ follow:

$$N_{i,t} = F_{i,t}(1 - s + sf(\theta_{i,t})), \quad U_{i,t} = sF_{i,t}(1 - f(\theta_{i,t}))$$

For each successful match, the household needs to pay the employment agency γ^b fraction of wage. We will describe the problem of employment agency in detail later. The household then has the following budget constraint:

$$B_{i,t} + P_{i,t}C_{i,t} = (1 + i_{t-1})B_{i,t} + \left(1 - s + s(1 - \gamma^b)f(\theta_{i,t})\right) W_{i,t}F_{i,t} + \Pi^T_{i,t} + \Pi^N_{i,t} + \Pi^E_{i,t}$$

where $B_{i,t}$ is a risk-free nominal bond denominated in time t unit of currency, i_t is the nonminal interest rate on the bond, $P_{i,t}$ is the price index for final good consumption basket $C_{i,t}$, $W_{i,t}$ is the nominal wage, and $\Pi^T_{i,t}$, $\Pi^N_{i,t}$, $\Pi^E_{i,t}$ are nominal profits for all tradeable and non-tradeable firms and employment agencies.

To summarize, the household's problem is:

$$\max_{C_{i,t}, F_{i,t}, B_{i,t}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_{i,t} - \chi_{i,t} \int_0^{F_{i,t}} f^{\omega} df \right)^{1-\sigma} \tag{4}$$

$$s.t. \quad B_{i,t} + P_{i,t}C_{i,t} = (1 + i_{t-1})B_{i,t} + \left(1 - s + s(1 - \gamma^b)f(\theta_{i,t})\right) W_{i,t}F_{i,t} + \Pi^T_{i,t} + \Pi^N_{i,t} + \Pi^E_{i,t} \tag{5}$$

The household takes prices and tightness as given and solves for optimal decisions on final good consumption $C_{i,t}$, labor force participation $F_{i,t}$ and saving $B_{i,t}$. The first order condition of this problem yields:

$$F^*_{i,t} = \left[\frac{1 - s + s(1-\gamma^b)f(\theta_{i,t})}{\chi_{i,t}} \frac{W_{i,t}}{P_{i,t}}\right] \tag{6}$$

$$1 = (1+i_t)\beta E_t\left[\frac{u_c(C_{i,t+1}, F_{i,t+1})}{u_c(C_{i,t}, F_{i,t})} \frac{P_{i,t}}{P_{i,t+1}}\right] \tag{7}$$

A.1.2 Firms

The final goods producer at location i purchases tradeable goods $Y^N_{i,t}$ and non-tradeable goods $Y^T_{i,t}$ and aggregate into final good $Y_{i,t}$ using a CES technology:

$$Y_{i,t} = \left(\phi_N^{\frac{1}{\eta}}(Y^N_{i,t})^{\frac{\eta-1}{\eta}} + \phi_T^{\frac{1}{\eta}}(Y^T_{i,t})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{8}$$

where η is the elasticity of substitution between tradeable and non-tradeable goods, ϕ_N and ϕ_T are the steady-state expenditure shares on tradeable and non-tradeable goods with $\phi_N + \phi_T = 1$. For tradeable goods, the final good producer at location i can choose to purchase tradeable goods from location j, denoted as $Y^{T,j}_{i,t}$, and aggregate them into tradeable goods at location i:[16]

$$Y^T_{i,t} = \left((\tau^i_{i,t})^{\frac{1}{\eta}}(Y^{T,j}_{i,t})^{\frac{\eta-1}{\eta}} + \sum_{j\neq i}^{N}(\tau^j_{i,t})^{\frac{1}{\eta}}(Y^{T,j}_{i,t})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{9}$$

The demand of location i firm for location j produced tradeable good is subject to shocks denoted by $\tau^j_{i,t}$, and we normalize $\sum_{j=1}^N \tau^j_{i,t} = 1$. For simplicity, we follow Hazell, Herreño, Nakamura, and Steinsson [2022] and assume no home bias in tradeable goods, e.g. $\tau^j_i = \tau^j$ for any $i = 1, 2, ..., N$.

In each location i, there is a continuum of monopolistic competitive intermediate goods producers in both tradeable and non-tradeable sectors. They are indexed by the variety z they produce and hire labor as their only inputs. Denote the non-tradeable variety z produced by the firm at location i as $Y_{i,t}(z)^N$ and tradeable variety z produced by the firm at location j purchased by location i as $Y^{T,j}_{i,t}$. The composite nontradeable goods $Y^N_{i,t}$ and tradeable goods produced at location j purchased by location i, $Y^{T,j}_{i,t}$, are given by CES

[16] Like in Hazell, Herreño, Nakamura, and Steinsson [2022], we assume that the elasticity of substitution between location i produced and location j produced tradeables is also η. The results qualitatively hold if we relax this assumption, but it greatly simplifies the form of the demand function.

aggregators:

$$Y_{i,t}^{N} = \left[\int_0^1 \left(Y_{i,t}^{N}(z)\right)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \tag{10}$$

$$Y_{i,t}^{T,j} = \left[\int_0^1 \left(Y_{i,t}^{T,j}(z)\right)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \tag{11}$$

Non-tradeable firms The technology for non-tradeable production is:

$$Y_{i,t}^{N,prod}(z) = Z_{i,t}^{N} N_{i,t}^{N}(z)$$

where $Z_{i,t}^{N}$ is the sectoral productivity shock and $N_{i,t}^{N}(z)$ is the firm's labor demand. Each firm z will set its price $P_{i,t}^{N}(z)$ under a Calvo fairy when it gets the chance to adjust the price with probability $1-\alpha$. They maximize the discounted expected flow of profit:

$$\max_{P_{i,t}^{N}(z)} E_t \sum_{k=0}^{\infty} \alpha^t \Theta_{i,t,t+k} (P_{i,t}^{N}(z) Y_{i,t+k}^{N}(z) - W_{i,t+k} Y_{i,t+k}^{N}(z)) \tag{12}$$

where $\Theta_{i,t,t+k}$ is the stochastic discounting factor, $P_{i,t}^{N}(z)$ is the price for non-tradeable good variety z produced at location i, $W_{i,t+k}$ is wage at location i, and $Y_{i,t+k}^{N}(z)$ is the demand for non-tradeable good variety z from cost minimization:

$$Y_{i,t}^{N}(z) = Y_{i,t}^{N} \left(\frac{P_{i,t}^{N}(z)}{P_{i,t}^{N}} \right)^{-\epsilon}$$

Tradeable firms Analogously to the non-tradeable sector, the intermediate firms also use a linear technology for production:

$$Y_t^{T,i}(z) = Z_{i,t}^{T} N_{i,t}^{T}(z)$$

Note that we use $Y_t^{T,i}(z)$ to denote the total production of intermediate good z produced in the tradeable sector at location i. In equilibrium, the production at location i equals the demand for this product at all locations:

$$Y_t^{T,i}(z) = \sum_{j=1}^{N} Y_{j,t}^{T,i}(z) \tag{13}$$

These firms are also subject to Calvo pricing and maximize:

$$\max_{P^T_{i,t}(z)} E_t \sum_{k=0}^{\infty} \alpha^t \Theta_{i,t,t+k}(P^T_{i,t}(z) Y^{T,i}_{t+k}(z) - W_{i,t+k} Y^{T,i}_{t+k}(z)) \tag{14}$$

Firm's Optimization The standard cost minimization problem of the final good producer is:

$$\begin{aligned}
\min_{\{Y^N_{i,t}, Y^{T,j}_{i,t}\}} \quad & Y^N_{i,t} P^N_{i,t} + \sum_{j=1}^{N} Y^{T,j}_{i,t} P^T_{j,t} \\
s.t. \quad & Y_{i,t} = \left(\phi_N^{\frac{1}{\eta}} (Y^N_{i,t})^{\frac{\eta-1}{\eta}} + \phi_T^{\frac{1}{\eta}} (Y^T_{i,t})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \\
& Y^T_{i,t} = \left((\tau^i_{i,t})^{\frac{1}{\eta}} (Y^{T,j}_{i,t})^{\frac{\eta-1}{\eta}} + \sum_{j \neq i}^{N} (\tau^j_{i,t})^{\frac{1}{\eta}} (Y^{T,j}_{i,t})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}
\end{aligned}$$

Solving this yields demands of $Y^N_{i,t}$ and $Y^{T,j}_{i,t}$ as functions of their prices, final good price, and desirable final good level:

$$Y^N_{i,t} = \phi_N Y_{i,t} \left(\frac{P^N_{i,t}}{P_{i,t}} \right)^{-\eta} \quad \text{(Demand for Non-tradeable)}$$

$$Y^{T,j}_{i,t} = \phi_T \tau^j_{i,t} Y_{i,t} \left(\frac{P^T_{j,t}}{P_{i,t}} \right)^{-\eta} \quad \text{(Demand for Tradeable produced at location } j\text{)}$$

Similarly, the cost minimization gives demands for the varieties $Y^N_{i,t}(z)$ and $Y^{T,j}_{i,t}(z)$ produced by intermediate good firms:

$$Y^N_{i,t}(z) = Y^N_{i,t} \left(\frac{P^N_{i,t}(z)}{P^N_{i,t}} \right)^{-\epsilon} \quad \text{(Demand for Non-tradeable variety)}$$

$$Y^{T,j}_{i,t}(z) = Y^{T,j}_{i,t} \left(\frac{P^T_{j,t}(z)}{P^T_{j,t}} \right)^{-\epsilon} \quad \text{(Demand for Tradeable variety produced at location } j\text{)}$$

Note that for tradeable goods produced at j, the prices are the same no matter at which location it is purchased. The price indices from cost minimizations are:

$$\begin{aligned}
P^N_{i,t} &= \left[\int_0^1 (P^N_{i,t}(z))^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}, \quad P^T_{i,t} = \left[\int_0^1 (P^T_{i,t}(z))^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}} \\
P_{i,t} &= \left[\phi_N (P^N_{i,t})^{1-\eta} + \phi_T \tau^i_{i,t} (P^T_{i,t})^{1-\eta} + \phi_T \tau^j_{i,t} (P^T_{j,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}
\end{aligned} \tag{15}$$

Intermediate firm's labor demand The optimal choice of labor by firm z is given by minimizing $W_{i,t}N_{i,t}^N$ subject to the demand firm z has:

$$Y_{i,t}^N \left(\frac{P_{i,t}^N(z)}{P_{i,t}^N} \right)^{-\epsilon} \leq Z_{i,t}^N N_{i,t}^N(z)$$

The optimality condition implies:

$$W_{i,t} = S_{i,t}^N(z) Z_{i,t}^N \tag{16}$$

where $S_{i,t}^N(z)$ is the firm's nominal marginal cost, which equals the Lagrangian multiplier of the previous problem. The intermediate firm sets price to maximize:

$$\max_{P_{i,t}^N(z)} E_t \sum_{k=0}^{\infty} \alpha^t \underbrace{\beta^t \frac{u'(C_{i,t+k})}{u'(C_{i,t})}}_{\equiv \Theta_{i,t,t+k}} (P_{i,t}^N(z) - S_{i,t+k}^N(z)) \underbrace{Y_{i,t+k}^N \left(\frac{P_{i,t}^N(z)}{P_{i,t+k}^N} \right)^{-\epsilon}}_{\equiv Y_{i,t+k}^N(z)} \tag{17}$$

The first order condition gives:

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[\Theta_{i,t,t+k} Y_{i,t+k}^N(z) \left(\frac{P_{i,t}^{N*}(z)}{P_{i,t-1}^N} - \frac{\epsilon}{\epsilon - 1} \underbrace{\frac{S_{i,t+k}^N(z)}{P_{i,t+k}^N}}_{\equiv MC_{i,t+k}^N(z)} \frac{P_{i,t+k}^N}{P_{i,t-1}^N} \right) \right] = 0 \tag{18}$$

where the $P_{i,t}^{N*}(z)$ is the optimal price firm z will set at time t when getting the chance. We define $MC_{i,t+k}^N(z) = S_{i,t+k}^N(z)/P_{i,t+k}^N$ as the *real marginal cost* of firm z and $\Theta_{i,t,t+k}$ is the stochastic discounting factor as the household owns the firms eventually.

The tradeable sector firms are also subject to Calvo pricing, and the optimal reset price is given by:

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[\Theta_{i,t,t+k} Y_{t+k}^{T,i}(z) \left(\frac{P_{i,t}^{T*}(z)}{P_{i,t-1}^T} - \frac{\epsilon}{\epsilon - 1} \underbrace{\frac{S_{i,t+k}^T(z)}{P_{i,t+k}^T}}_{\equiv MC_{i,t+k}^T(z)} \frac{P_{i,t+k}^T}{P_{i,t-1}^T} \right) \right] = 0 \tag{19}$$

Recall that $P_{i,t}^T(z)$ and $P_{i,t}^T$ denote the prices of tradeable goods for firm z or in aggregate produced at location i. $S_{i,t}^T(z)$ is again the nominal marginal cost of tradeable goods firm z with:

$$W_{i,t} = S_{i,t}^T(z) Z_{i,t}^T$$

A.1.3 Employment Agency

In each location i there is a representative agency. It charges a fee proportional to the salary of a match, $\gamma^b W_{i,t} M_{i,t}$, and it pays a cost in terms of final consumption goods, $\gamma^c V_{i,t}$, for vacancies $V_{i,t}$ it posts. The employment agency takes real wage ($w_{i,t} = W_{i,t}/P_{i,t}$) and unemployment at the start of the period ($U^s_{i,t}$) as given and maximizes real profit by choosing $V_{i,t}$, which is equivalent to choosing tightness $\theta_{i,t}$. We then follow Benigno and Eggertsson [2023] to assume that wage settings can be *flexible* or *rigid*. When the labor market is tight enough, i.e. $\theta_{i,t} > 1$, wage is flexible and follows from the solution of the employment agency's profit optimization problem:

$$\max_{\theta_{i,t}} \quad \gamma^b w_{i,t} U^s_{i,t} f(\theta_{i,t}) - \gamma^c U^s_{i,t} \theta_{i,t} \tag{20}$$

This yields the relation between flexible wages and tightness:

$$w^{flex}_{i,t} = \frac{\gamma^c}{m_{i,t}(1-\mu)\gamma^b} \theta^{\mu}_{i,t} \tag{21}$$

When wage is set flexibly, it increases in tightness. To keep things simple, we follow Gitti [2024] and write:

$$w_{i,t} = \begin{cases} w^{flex}_{i,t} & \theta_{i,t} > 1 \\ (\bar{w}_i)^{\delta} (w^{flex}_{i,t})^{1-\delta} & \theta_{i,t} \leq 1 \end{cases} \tag{22}$$

where $\bar{w}_i$ is the steady state level of real wage at location i that will be derived in Appendix A.3. $0 \leq \delta \leq 1$ governs how quickly the rigid wage rate adjusts to its flexible rate when the labor market is slack.

A.1.4 Equilibrium

In equilibrium, all the intermediate goods firms in tradeable and non-tradeable sectors have productions that match their demands. In the final goods market, we have the total production of final goods at location i equals final goods consumption chosen by the household plus the cost paid for posting vacancies.

$$Y_{i,t} = C_{i,t} + \gamma^c V_{i,t}$$

The labor market clears at local level with:

$$N_{i,t} = F_{i,t}(1 - s + sf(\theta_{i,t})) = \int_0^1 N_{i,t}^N(z)dz + \int_0^1 N_{i,t}^T(z)dz$$

where $F_{i,t}$ is the supply of labor force chosen by the household and $\theta_{i,t}$ is the local labor market tightness chosen by the employment agency. Moreover, the optimal labor supply from the household, Equation (6), gives the relationship between $F_{i,t}$ and $\theta_{i,t}$. Finally, the wage setting equation (22) gives the relation between $\theta_{i,t}$ and wage. The labor demands $N_{i,t}^N(z)$ and $N_{i,t}^T(z)$ are determined by wage through its impact on the real marginal cost of the firms.

A.2 List of Key Equations

We summarize the list of key equations in our model. The household's optimal choices are:

$$F_{i,t}^* = \left[\frac{1 - s + s(1-\gamma^b)f(\theta_{i,t})}{\chi_{i,t}} \frac{W_{i,t}}{P_{i,t}}\right] \tag{23}$$

$$1 = (1 + i_t)\beta E_t \left[\frac{u_c(C_{i,t+1}, F_{i,t+1})}{u_c(C_{i,t}, F_{i,t})} \frac{P_{i,t}}{P_{i,t+1}}\right] \tag{24}$$

The employment and unemployment levels are pinned down by matching:

$$N_{i,t} = F_{i,t}(1 - s + sf(\theta_{i,t})), \quad U_{i,t} = sF_{i,t}(1 - f(\theta_{i,t})), \quad f(\theta_{i,t}) = m_{i,t}\theta_{i,t}^{1-\mu} \tag{25}$$

The nominal marginal costs of non-tradeable and tradeable firms:

$$W_{i,t} = S_{i,t}^N(z)Z_{i,t}^N, \quad W_{i,t} = S_{i,t}^T(z)Z_{i,t}^T \tag{26}$$

The real marginal costs:

$$MC_{i,t}^N(z) = \frac{S_{i,t}^N(z)}{P_{i,t}^N}, \quad MC_{i,t}^T(z) = \frac{S_{i,t}^N(z)}{P_{i,t}^T} \tag{27}$$

The optimal price adjustments:

$$\sum_{k=0}^{\infty}\alpha^k E_t\left[\Theta_{i,t,t+k}Y_{i,t+k}^N(z)\left(\frac{P_{i,t}^{N*}(z)}{P_{i,t-1}^N} - \frac{\epsilon}{\epsilon-1}MC_{i,t+k}^N(z)\frac{P_{i,t+k}^N}{P_{i,t-1}^N}\right)\right] = 0 \tag{28}$$

$$\sum_{k=0}^{\infty}\alpha^k E_t\left[\Theta_{i,t,t+k}Y_{t+k}^{T,i}(z)\left(\frac{P_{i,t}^{T*}(z)}{P_{i,t-1}^T} - \frac{\epsilon}{\epsilon-1}MC_{i,t+k}^T(z)\frac{P_{i,t+k}^T}{P_{i,t-1}^T}\right)\right] = 0 \tag{29}$$

The wage setting equation:

$$w_{i,t} = \begin{cases} w_{i,t}^{flex} & \theta_{i,t} > 1 \\ (\bar{w}_i)^{\delta}(w_{i,t}^{flex})^{1-\delta} & \theta_{i,t} \leq 1 \end{cases} \tag{30}$$

The flexible wage from employment agency's optimality condition:

$$w_{i,t}^{flex} = \frac{\gamma^c}{m_{i,t}(1-\mu)\gamma^b}\theta_{i,t}^{\mu} \tag{31}$$

Finally, the CES aggregators for production (8)-(11) and definitions of prices (15).

A.3 Steady State

We consider the steady state with zero inflation, balanced trade as in Hazell, Herreño, Nakamura, and Steinsson [2022] . The steady state is derived under flexible wage as in Benigno and Eggertsson [2023].

Price setting In steady state, productivities $Z_i^N = Z_i^T = 1$, each intermediate firm has the same nominal marginal costs tied to nominal wage. From (26) we have:

$$W_i = S_i^N = S_i^T$$

All the relative prices are 1. The steady states from (28) and (29) imply:

$$\frac{W_i}{P_i^N} = \frac{W_i}{P_i^T} = \frac{\epsilon - 1}{\epsilon} = \frac{W_i}{P_i}$$

Note that $\bar{w}_i$ in wage setting equation (30) equals to steady state level W_i/P_i.

Household and Labor Market The steady state level of intertemporal Euler Euquation (24) implies $1 = (1+i)\beta$. The four key variables in the labor market are $\{N_i, F_i, \theta_i, w_i\}$ which are determined by the steady state versions of (23), (25), (31), and the real wage equation above:

$$\begin{aligned} F_i &= \left[\frac{1 - s + s(1-\gamma^b)f(\theta_i)}{\chi_i}\frac{W_i}{P_i}\right] \\ N_i &= F_i(1 - s + sf(\theta_i)) \\ \frac{W_i}{P_i} &= \frac{\gamma^c}{m_i(1-\mu)\gamma^b}\theta_i^{\mu} \end{aligned}$$

Production The demand curves of tradeable and non-tradeable goods imply:

$$Y_i^N = \phi_N Y_i, \quad Y_i^{T,j} = \phi_T \tau_i^j Y_i \tag{32}$$

Denote $N_{i,t}^N \equiv \int_0^1 N_{i,t}^N(z)dz$ and $N_{i,t}^T \equiv \int_0^1 N_{i,t}^T(z)dz$ and in equilibrium using non-tradeable goods production we have:

$$N_i^N = Y_i^N = \phi_N Y_i$$

Similarly, for tradeable goods:

$$N_i^T = Y^{T,i} = \sum_{j=1}^{N} Y_j^{T,i} = \phi_T \sum_{j=1}^{N} \tau_j^i Y_j = \phi_T \tau^i \sum_{j=1}^{N} Y_j$$

where the second equality uses (13) and the last equality uses no-home-bias assumption, $\tau_j^i = \tau^i$ for any $j = 1, ..., N$. In equilibrium, $Y_i = C_i + \gamma^c V_i$ and $V_i = U_i f(\theta_i)$.

A.4 Derivation of Sectoral Phillips Curve

Denote hat variables as the log differences from steady states. Define the non-tradeable/tradeable goods inflation at location i and time t as:

$$\pi_{i,t}^K = p_{i,t}^K - p_{i,t-1}^K, \quad K = T, N \tag{33}$$

where p denotes the log prices. The first order Taylor expansion of Equation (28) and (29) around the steady state gives:

$$\begin{aligned} p_{i,t}^{N*} - p_{i,t-1}^N &= (1-\alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t[\widehat{mc}_{i,t+k}^N + (p_{i,t+k}^N - p_{i,t-1}^N)] \\ &= (1-\alpha\beta)\widehat{mc}_{i,t}^N + \pi_{i,t}^N + \alpha\beta E_t[p_{i,t+1}^{N*}(z) - p_{i,t}^N] \end{aligned} \tag{34}$$

Using (15) and the fact that a random set of firms change prices every period and all of them will set the same price, we have:

$$(P_{i,t}^N)^{1-\epsilon} = \alpha (P_{i,t-1}^N)^{1-\epsilon} + (1-\alpha)(P_{i,t}^{N*})^{1-\epsilon}$$

Linearize this equation we get:

$$p_{i,t}^N = \alpha p_{i,t-1}^N + (1-\alpha) p_{i,t}^{N*}$$

Combine with (33):

$$\pi_{i,t}^{N} = (1-\alpha)(p_{i,t}^{N*} - p_{i,t-1}^{N})$$

Plug this into (34) we get the linearized Phillips curve for non-tradeable sector:

$$\pi_{i,t}^{N} = \beta E_t \pi_{i,t+1}^{N} + \lambda \widehat{mc}_{i,t}^{N} \tag{35}$$

where $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. Analogously we can derive the linearized Phillips curve for tradeable sector:

$$\pi_{i,t}^{T} = \beta E_t \pi_{i,t+1}^{T} + \lambda \widehat{mc}_{i,t}^{T} \tag{36}$$

Now linearize (27) we have:

$$\widehat{mc}_{i,t}^{K} = s_{i,t}^{K} - p_{i,t}^{K}, \quad K = N, T$$

Combine with the linearized (26)

$$\widehat{w}_{i,t} = s_{i,t}^{N} + z_{i,t}^{N} = s_{i,t}^{T} + z_{i,t}^{T}$$

We get:

$$\widehat{mc}_{i,t}^{K} = \widehat{w}_{i,t} - p_{i,t}^{K} - z_{i,t}^{K}, \quad K = H, N \tag{37}$$

where $-z_{i,t}^{K}$ is cost push shock for sector $K = N, T$, and $\widehat{w}_{i,t}$ is the deviation of **nominal wage** from steady state.

We now log-linearize the real wage process in (30). First, we log-linearize the flexible wage Equation (31):[17]

$$\widehat{w}_{i,t}^{flex} = \mu \widehat{\theta}_{i,t} + \widehat{\nu}_{i,t}$$

where $\widehat{\nu}_{i,t} = -\widehat{m}_{i,t}$ captures shocks to matching technology. The linearized wage setting is

[17] Note that in our simple baseline, we treat γ^c and γ^b as exogenous parameters as in Benigno and Eggertsson [2023]. One can easily extend it to the possibility that $\gamma_{i,t}^{c}$ and $\gamma_{i,t}^{b}$ represent shocks to the costs and benefits of the employment agency, as considered in Gitti [2024]. In that case:

$$\widehat{w}_{i,t}^{flex} = \mu \widehat{\theta}_{i,t} + \underbrace{\gamma_{i,t}^{c} - \gamma_{i,t}^{b} - \widehat{m}_{i,t}}_{\equiv \widehat{\nu}_{i,t}}$$

where $\widehat{\nu}_{i,t}$ can capture shocks to employment agency and the matching technology.

then:

$$\widehat{w}_{i,t} - p_{i,t} = \begin{cases} \mu\widehat{\theta}_{i,t} - \widehat{m}_{i,t} & \text{if } \widehat{\theta}_{i,t} > 1 \\ (1-\delta)(\mu\widehat{\theta}_{i,t} + \widehat{\nu}_{i,t}) & \text{if } \widehat{\theta}_{i,t} \leq 1 \end{cases} \tag{38}$$

Note the local labor market is unified, not sector-specific. Employed at any sector K would get the same wage. Plug this into (37) and combine with (35) and (36), we get for each sector K (tradeable or non-tradeable):

$$\pi_{i,t}^K = \beta E_t \pi_{i,t+1}^K + \lambda(1-\delta)\mu\widehat{\theta}_{i,t} + \lambda\delta\mu\widehat{\theta}_{i,t}\mathbb{1}(\widehat{\theta}_{i,t} > 1) + \lambda(1-\delta)\widehat{\nu}_{i,t} + \lambda\delta\mathbb{1}(\widehat{\theta}_{i,t} > 1)\widehat{\nu}_{i,t} - \lambda\widehat{p}_{i,t}^K - \lambda z_{i,t}^K \tag{39}$$

We can simplify the coefficients and get the sector K Phillips curve:

$$\pi_{i,t}^K = \beta E_t \pi_{i,t+1}^K + \gamma_0\widehat{\theta}_{i,t} + \gamma_1\widehat{\theta}_{i,t}\mathbb{1}(\widehat{\theta}_{i,t} > 1) + \kappa_0\widehat{\nu}_{i,t} - \lambda\widehat{p}_{i,t}^K - \lambda z_{i,t}^K \tag{40}$$

where:

$$\gamma_0 = \lambda\mu(1-\delta) \tag{41}$$

$$\gamma_1 = \lambda\delta\mu \tag{42}$$

$$\kappa_0 = \lambda(1-\delta) + \lambda\delta\mathbb{1}(\widehat{\theta}_{i,t} > \widehat{\theta}_i^*) \tag{43}$$

$$\widehat{p}_{i,t}^K = p_{i,t}^K - p_{i,t}, \quad K = N, T \tag{44}$$

where $\widehat{p}_{i,t}^K$ denotes the regional price of sector K relative to aggregate price in region i.

A.5 Regional inflation all sectors

As we do not observe regional tradeable or non-tradeable prices nor these expectations, we need to derive the regional all-sector inflation. First we linearize the aggregate price index in (15):

$$p_{i,t} = \phi_N p_{i,t}^N + \phi_T \tau_i^i p_{i,t}^T + \phi_T \sum_{j \neq i}^{N} \tau_i^j p_{j,t}^T, \quad \forall i = 1, ..., N \tag{45}$$

It implies the following relationship between aggregate and sectoral inflations:

$$\pi_{i,t} = \phi_N \pi_{i,t}^N + \phi_T \sum_{j=1}^{N} \tau_i^j \pi_{j,t}^T \tag{46}$$

Following the similar rationale, we can write regional expected inflation:

$$E_t\pi_{i,t+1} = \phi_N E_t\pi^N_{i,t+1} + \phi_T \sum_{j=1}^{N} \tau_i^j E_t\pi^T_{j,t+1} \tag{47}$$

Moreover, as $\tau_i^j = \tau^j$, this means we can write:

$$E_t\pi^T_{t+1} \equiv \sum_{j=1}^{N} \tau^j E_t\pi^T_{j,t+1} \tag{48}$$

This is a time-varying belief about tradeable goods prices that is *common across locations* i. This means that depending on ϕ_T (and relative variations of $E_t\pi^T_{t+1}$ to local $\pi^N_{i,t+1}$), the expectations may have most variations from time series dimension. We can then re-write the local all-sector inflation (46), using the regional Phillips curves (40):

$$\begin{aligned}
\pi_{i,t} &= \beta E_t\pi_{i,t+1} + \phi_N[\gamma_0\widehat{\theta}_{i,t} + \gamma_1\widehat{\theta}_{i,t}\mathbb{1}_{i,t}] + \phi_N\kappa_0\widehat{\nu}_{i,t} - \lambda\phi_N z^N_{i,t} \\
&\quad + \phi_T \sum_{j=1}^{N} \tau^j \left(\gamma_0\widehat{\theta}_{j,t} + \gamma_1\widehat{\theta}_{j,t}\mathbb{1}_{j,t} + \kappa_0\widehat{\nu}_{j,t} - \lambda z^T_{j,t}\right) - \lambda[\phi_N\widehat{p}^N_{i,t} + \phi_T \sum_{j=1}^{N} \tau^j \widehat{p}^T_{j,t}] \\
&= \beta E_t\pi_{i,t+1} + \phi_N[\gamma_0\widehat{\theta}_{i,t} + \gamma_1\widehat{\theta}_{i,t}\mathbb{1}_{i,t}] + \phi_N\kappa_0\widehat{\nu}_{i,t} - \lambda\phi_N z^N_{i,t} + \phi_T X_t - \lambda(1-\phi_N)p_{i,t}
\end{aligned} \tag{49}$$

where

$$X_t = \sum_{j=1}^{N} \tau^j \left(\gamma_0\widehat{\theta}_{j,t} + \gamma_1\widehat{\theta}_{j,t}\mathbb{1}_{j,t} + \kappa_0\widehat{\nu}_{j,t} - \lambda z^T_{j,t} + \lambda p_{j,t}\right)$$

and the last equality follows from (45):

$$\begin{aligned}
\phi_N\widehat{p}^N_{i,t} + \phi_T \sum_j \tau^j \widehat{p}^T_{j,t} &= \phi_N p^N_{i,t} + \phi_T \sum_j \tau^j p^T_{j,t} - \phi_N p_{i,t} - \phi_T \sum_j \tau^j p_{j,t} \\
&= (1-\phi_N)p_{i,t} - \phi_T \sum_j \tau^j p_{j,t}
\end{aligned}$$

Now notice that we cannot directly run (49) because $p_{i,t}$ is correlated with all the shocks to $\pi_{i,t}$, we need to move it to the other side:

$$\begin{aligned}
\pi_{i,t} = &\frac{\beta}{1+\lambda(1-\phi_N)} E_t\pi_{i,t+1} + \frac{\phi_N}{1+\lambda(1-\phi_N)}[\gamma_0\widehat{\theta}_{i,t} + \gamma_1\widehat{\theta}_{i,t}\mathbb{1}_{i,t}] \\
&- \frac{\lambda(1-\phi_N)}{1+\lambda(1-\phi_N)} p_{i,t-1} + \frac{\phi_T}{1+\lambda(1-\phi_N)} X_t + \epsilon_{i,t}
\end{aligned} \tag{50}$$

Now omitting X_t will over estimate γ_0 and γ_1 as (i) in X_t there is $\widehat{\theta}_{i,t}$; (ii) if $\widehat{\theta}_{j,t}$ is highly correlated with $\theta_{i,t}$, which seems true in data. Note that according to (41)-(43), if λ or μ (sensitivity of wage to tightness) is close to 0, the estimated slope would still be quite close to 0, and the Phillips curve would be flat.

B First stage F-statistics

We report the first-stage F-statistics from regressing log tightness and its interaction term with the dummy variable on our instruments (Table 2). Let $\Delta S_{i,t}$ denote the Shift-Share IV. Table B.1 shows the results, indicating a strong set of instruments.

Table B.1: First Stage Results

	$\ln \theta_{i,t}$	$\ln \theta_{i,t} \times \mathbb{1}_{i,t}$
$\Delta S_{i,t}$	11.60***	0.09
	(0.84)	(0.48)
$\Delta S_{i,t} \times \mathbb{1}_{i,t}$	5.33***	3.22***
	(0.331)	(0.19)
Observations	2175	2175
MSA F.E.	Y	Y
Time F.E.	Y	Y
F-stat	244.71	148.25

Notes: We estimate the first stage regressions $y_{i,t} = \beta_0 + \beta_1 \Delta S_{i,t} + \beta_2 \Delta S_{i,t} \times \mathbb{1}_{i,t} + \beta_{D\times\theta} + \alpha_i + \Theta_t + \varepsilon_{i,t}$. $y_{i,t}$ *are the instrumented variables* $\ln \theta_{i,t}$ *and* $\ln \theta_{i,t} \times \mathbb{1}_{i,t}$. *Sample is 2000Q3-2024Q3.*

C Regional Estimation with Monthly Data

In this appendix, we demonstrate that our baseline results remain robust when using monthly frequency data, as in [Gitti 2024]. [18] Table C.1 replicates the baseline OLS estimations from Table 2 using monthly data, while Table 3 replicates the baseline IV estimations from Table 3 under the same adjustment. One limitation of using monthly data is that, for most MSAs, CPI indices are only available bi-monthly. Following [Gitti 2024], we address this by linearly interpolating the missing CPI indices.

[18] The key difference between our approach and that of [Gitti 2024] is that we control for a full set of year-month fixed effects, guided by our theoretical framework, whereas [Gitti 2024] includes year-quarter fixed effects and their interactions with MSA fixed effects. Additionally, our theoretical formulation of the regional Phillips Curve differs from that of [Gitti 2024].

Table C.1: Estimation of Iterated Forward (2) with OLS, Monthly Data

	Month-to-month				Year-to-year			
	Core CPI		HL CPI		Core CPI		HL CPI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_θ	1.01***	1.39***	0.92***	0.89***	0.85***	1.12***	1.06***	0.77***
	(0.130)	(0.279)	(0.176)	(0.307)	(0.041)	(0.085)	(0.053)	(0.077)
$\beta_{D\times\theta}$	2.47***	-0.40	3.15***	-0.32	3.50***	0.00	4.77***	0.06
	(0.415)	(0.513)	(0.566)	(0.563)	(0.128)	(0.154)	(0.170)	(0.139)
β_D	-0.08	-0.04	0.07	0.07	-0.15**	-0.10	-0.15*	-0.12**
	(0.221)	(0.222)	(0.299)	(0.245)	(0.070)	(0.068)	(0.092)	(0.062)
$p_{i,t-1}$	-0.00	-0.00**	-0.00***	-0.01***				
	(0.000)	(0.002)	(0.001)	(0.002)				
$p_{i,t-12}$					-0.00**	-0.04***	-0.03***	-0.05***
					(0.002)	(0.006)	(0.002)	(0.006)
Observations	4651	4651	4651	4651	4420	4420	4420	4420
MSA F.E.	Y	Y	Y	Y	Y	Y	Y	Y
Time F.E.	N	Y	N	Y	N	Y	N	Y

Notes: We estimate $\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log\theta_{i,t} + \beta_{D\times\theta}D_{i,t} \times \log\theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + \Theta_t + \varepsilon_{i,t}$ by iterating expected inflation forward and assume common across MSAs long-run expectation anchored by monetary policy as in [Hazell, Herreño, Nakamura, and Steinsson 2022]. Columns (1) to (4) use month-to-month CPI, Columns (5) to (8) use year-to-year CPI at monthly frequency. To avoid the endogeneity issue we include $p_{i,t-12}$ instead of $p_{i,t-1}$ when using year-to-year inflation. Sample is 2000M7-2024M7.

Table C.2: Estimation of Iterated Forward (2) with IV, Monthly Data

	Month-to-month				Year-to-year			
	Core CPI		HL CPI		Core CPI		HL CPI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_θ	0.70**	2.34**	-0.10	1.04	0.99***	2.35***	0.51**	1.97***
	(0.308)	(1.158)	(0.413)	(1.233)	(0.086)	(0.395)	(0.256)	(0.337)
$\beta_{D\times\theta}$	35.95***	-9.12*	48.19***	-5.34	11.42***	-1.67	37.86***	-1.42
	(7.080)	(4.954)	(9.517)	(5.325)	(2.081)	(1.420)	(5.955)	(1.282)
β_D	-9.66***	1.00	-11.99***	0.74	-2.80***	-0.14	-9.99***	-0.19
	(2.018)	(0.853)	(2.699)	(0.934)	(0.627)	(0.252)	(1.815)	(0.229)
$p_{i,t-1}$	-0.01***	-0.00	-0.01***	-0.01**				
	(0.002)	(0.002)	(0.003)	(0.003)				
$p_{i,t-12}$					-0.02***	-0.02**	-0.10***	-0.03***
					(0.005)	(0.010)	(0.014)	(0.009)
Observations	4296	4296	4296	4296	4230	4230	4230	4230
Weak ID Test	18.394	18.803	18.528	18.365	15.512	18.752	17.116	18.651
MSA F.E.	Y	Y	Y	Y	Y	Y	Y	Y
Time F.E.	N	Y	N	Y	N	Y	N	Y

Notes: We estimate $\pi_{i,t} = \beta_{\pi^e}\pi^e_{i,t+1} + \beta_\theta \log\theta_{i,t} + \beta_{D\times\theta} D_{i,t} \times \log\theta_{i,t} + \beta_D D_{i,t} + \beta_p p_{i,t-1} + \alpha_i + \Theta_t + \varepsilon_{i,t}$ *by iterating expected inflation forward and assume common across MSAs long-run expectation anchored by monetary policy as in [Hazell, Herreño, Nakamura, and Steinsson 2022]. Both* $\log\theta_{i,t}$ *and* $D_{i,t} \times \log\theta_{i,t}$ *are instrumented with shift-share IV and its product with* $D_{i,t}$*. Columns (1) to (4) use month-to-month CPI, Columns (5) to (8) use year-to-year CPI at monthly frequency. To avoid the endogeneity issue we include* $p_{i,t-12}$ *instead of* $p_{i,t-1}$ *when using year-to-year inflation. Sample is 2000M7-2024M7.*

D Transforming Year-to-Year Inflation Expectations into Quarter-to-Quarter Ones

The various inflation expectations we use are one-year-ahead. For example, in the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: "*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*" The answer to this question is then the one-year-ahead inflation expectation $E_t\pi_{t+4,t}$. To keep consistency with the quarter-to-quarter inflation when estimating a New-Keynesian Phillips curve, we rescale the one-year-ahead expected inflation in the following way.[19]

We first assume that realized quarter-to-quarter inflation follows an AR(1) process with persistence ρ_π:

$$\pi_{t+1,t} = \rho_\pi \pi_{t,t-1} + \epsilon_t \tag{D.1}$$

Consumers may or may not have the correct belief on ρ_π. We assume they believe that persistence is $\widetilde{\rho}$, so that the perceived law of motion of inflation is

$$\pi_{t+1,t} = \widetilde{\rho}\pi_{t,t-1} + \epsilon_t \tag{D.2}$$

Consumers observe a noisy signal on inflation: $s_t = \pi_{t,t-1} + \eta_t$ where η_t is of mean zero, i.i.d., orthogonal to ϵ_t and independent across time. Consumers will form quarter-to-quarter inflation expectation, denoted by $E_t\pi_{t+1,t}$, using a Kalman filter:

$$E_t\pi_{t+1,t} = \widetilde{\rho}E_t\pi_{t,t-1} = \widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t \tag{D.3}$$

where K is the Kalman gain.

We do observe one-year-ahead expected inflation:

$$E_t\pi_{t+4,t} \equiv E_t(\pi_{t+4,t+3} + \pi_{t+3,t+2} + \pi_{t+2,t+1} + \pi_{t+1,t})$$

Using the perceived law of motion (D.2):

$$\begin{aligned} E_t\pi_{t+4,t} &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)E_t\pi_{t+1,t} \\ &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)(\widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t) \end{aligned} \tag{D.4}$$

[19] For details of this approach extended to multi-variable joint learning environment, see Hou [2020].

We use the $t-1$ version of (D.4) and plug it in the above equation to obtain:

$$E_t\pi_{t+4,t} = \underbrace{\widetilde{\rho}(1-K)}_{\psi_1} E_{t-1}\pi_{t+3,t-1} + \underbrace{(1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K}_{\psi_2}\pi_{t,t-1} + (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K\eta_t \quad \text{(D.5)}$$

We can estimate equation (D.5) with OLS because η_t is the i.i.d noise orthogonal to inflation. We need to use quarter-to-quarter (not annualized) inflation for $\pi_{t,t-1}$ and year-ahead expected inflation and its lag from the Michigan Survey of Consumers. We use Headline CPI as proxy for $\pi_{t,t-1}$.

Given the estimate on the perceived persistence of inflation, the quarter-to-quarter expected inflation is implied by equation (D.4):

$$E_t\pi_{t+1,t} = \frac{1}{1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3} E_t\pi_{t+4,t} \quad \text{(D.6)}$$

E Data Appendix

E.1 Aggregate Data

Supply shocks v: As in Benigno and Eggertsson [2023], we proxy supply shocks v constructed as the four-quarter average of the principal component of the following three series: headline shocks, both to CPI and PCE, and import shock. The CPI or PCE headline shock is the difference between the annualized quarterly inflation rate computed using the CPI or PCE price index and that computed using the CPI or PCE price index excluding energy and food. The import shock is the difference between the annualized quarterly inflation rate computed using the import-price deflator and that computed using the GDP deflator. All series are obtained from the FRED database (https://fred.stlouisfed.org).

Labour Market tightness θ: Computed in Michaillat and Saez [2024], obtained from https://pascalmichaillat.org/13/.

Core inflation π: "CPILFES", obtained from from the FRED database (https://fred.stlouisfed.org).

Inflation expectations: We use one-year-ahead inflation expectations. "Michigan" is the inflation expectations series of the Surveys of Consumers published by the University of Michigan (http://www.sca.isr.umich.edu). We use the median expectations on samples starting after 1978, and the mean one for longer samples. “Cleveland ” is the inflation expectations series published by the Federal Reserve Bank of Cleveland (https://www.clevelandfed.org/indicators-and-data/inflation-expectations), “SPF” is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters), “Livingston” is the Livingston Survey published by the Federal Reserve Bank of Philadelphia (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-survey), “SCE” is the Survey of Consumer Expectations series published by the Federal Reserve Bank of New York (https://www.newyorkfed.org/microeconomics/sce#/), “SoFIE” is the Survey of Firms’ Inflation Expectations series published by the Federal Reserve Bank of Cleveland (https://www.clevelandfed.org/indicators-and-data/survey-of-firms-inflation-expectations), “BIE” is the Business Inflation Expectations survey published by the Federal Reserve Bank of Atlanta (https://www.atlantafed.org/research/inflationproject/bie).

Monetary policy shocks: constructed by Bu, Rogers, and Wu [2021], updated series downloaded from the web page of Chunya Bu (https://sites.google.com/view/chunyabu).

E.2 MSA Data

Labour Market Tightness: unemployment numbers at the MSA level and vacancy numbers at the State level are obtained from Job Openings and Labor Turnover Survey (JOLTS) of U.S. Bureau of Labor Statistics (BLS): https://www.bls.gov/jlt/. The weights used to compute the MSA level vacancies are from U.S. Census 2020 available through IPUMS (https://usa.ipums.org/usa/).

Inflation: Headline CPI is all items for all urban consumers and Core CPI is all items less food and energy for all urban consumers at MSA levels. Both are available from the BLS (https://www.bls.gov/data/).

Inflation expectations: We use the median of density forecast of 1 year ahead inflation expectation (*Q9_cent50* in the public micro data) from the micro data of Survey of Consumer Expectation (SCE: https://www.newyorkfed.org/microeconomics/sce#/). The SCE contains commuting zone information for survey respondents. We perform a cross-walk from the commuting zone to the county using information from USDA (https://www.ers.usda.gov/data-products/commuting-zones-and-labor-market-areas), then from county to MSAs using information from the Quaterly Census of Employment and Wages (QCEW) of BLS (https://www.bls.gov/cew/classifications/areas/area-guide.htm).

Shift-share IV: The shift-share IV is constructed as:

$$z_{i,t} = \sum_x \bar{S}_{x,i} \Delta_{3Y} log(S_{x,t})$$

$\bar{S}_{x,i}$ is employment share of industry x in MSA i, obtained from the 2000 Census. $S_{x,t}$ is the national employment levels of industry x at time t obtained from the Current Population Survey (CPS) 2000-2024. Both the Census and CPS are available through IPUMS (https://usa.ipums.org/usa/).

F Aggregate Estimation Results with Different Samples

Table F.1: Estimation of Equation (3), OLS, Post Volcker

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	0.64**	0.58**	0.61**	0.61**	0.80***	0.88***	0.94***	1.02***
	(0.26)	(0.26)	(0.26)	(0.26)	(0.17)	(0.20)	(0.20)	(0.20)
β_θ	0.44*	0.50**	0.35	0.38	0.81***	0.84***	0.64**	0.73***
	(0.25)	(0.24)	(0.28)	(0.29)	(0.23)	(0.22)	(0.24)	(0.25)
$\beta_{\theta \times D}$	3.26***	3.26***	2.69**	3.04**	1.41	1.20	0.14	1.13
	(1.09)	(1.04)	(1.20)	(1.51)	(1.09)	(1.10)	(1.25)	(1.38)
β_v		-0.01	-0.02	-0.01		-0.04**	-0.05**	-0.05**
		(0.02)	(0.02)	(0.02)		(0.02)	(0.02)	(0.02)
$\beta_{v \times D}$		0.17***	0.18***	0.18***		0.11*	0.12**	0.13**
		(0.06)	(0.06)	(0.06)		(0.06)	(0.05)	(0.06)
β_D			0.47	0.45			0.74*	0.68
			(0.49)	(0.50)			(0.44)	(0.44)
$\beta_{\pi_{-1}}$				-0.06				-0.20
				(0.14)				(0.13)
N	64	64	64	64	64	64	64	64
adj. R^2	0.59	0.63	0.63	0.62	0.67	0.69	0.70	0.71

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$. *Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for* $\log\theta_t > 0$ *and v is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 2008Q1-2023Q4.*

Table F.2: Estimation of Equation (3), IV Using Monetary Shock as an Instrument, Post Volcker

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	1.07***	0.80*	0.72**	0.91**	0.84***	1.08***	0.97***	1.27***
	(0.40)	(0.42)	(0.31)	(0.40)	(0.20)	(0.31)	(0.21)	(0.35)
β_θ	0.09	0.23	-0.07	-0.22	0.44**	0.62***	0.31	0.30
	(0.25)	(0.15)	(0.25)	(0.30)	(0.19)	(0.13)	(0.28)	(0.36)
$\beta_{\theta\times D}$	4.97	4.37*	1.58	1.28	3.78	1.21	0.82	1.46
	(4.07)	(2.27)	(2.54)	(3.25)	(2.72)	(2.06)	(1.69)	(2.36)
β_D			1.24	2.16			0.90	1.44
			(1.14)	(1.54)			(0.93)	(1.19)
β_v		-0.01*	-0.01	-0.02		-0.06***	-0.05***	-0.07***
		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.02)
$\beta_{v\times D}$		0.16***	0.20***	0.28***		0.08***	0.11***	0.16***
		(0.03)	(0.05)	(0.10)		(0.03)	(0.04)	(0.06)
$\beta_{\pi_{-1}}$				-0.32*				-0.46***
				(0.16)				(0.16)
Observations	64	64	64	64	64	64	64	64

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$. *Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for* $\log\theta_t > 0$ *and v is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* π^e_{t+1}, $\log\theta_t$ *and* π_{t-1} *are instrumented by six lags of a measure of monetary shocks and six lags of their square. The measure of monetary shocks is the updated series of Bu, Rogers, and Wu [2021], as kindly provided by the authors. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 2008Q1-2023Q4.*

Table F.3: Estimation of Equation (3), OLS, Long Sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	1.09***	1.03***	0.77***	0.77***	0.97***	0.97***	0.98***	0.58***
	(0.04)	(0.04)	(0.07)	(0.07)	(0.04)	(0.04)	(0.04)	(0.06)
β_θ	0.17	0.10	0.05	0.05	0.39*	0.38*	0.26	0.36*
	(0.19)	(0.19)	(0.19)	(0.19)	(0.20)	(0.21)	(0.22)	(0.19)
$\beta_{\theta\times D}$	4.11***	4.33***	0.97	0.97	-1.05	-1.01	-2.16*	-2.55**
	(0.83)	(0.80)	(1.03)	(1.03)	(0.88)	(0.90)	(1.18)	(1.02)
β_v		0.05***	0.04***	0.04***		0.00	0.00	-0.00
		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)
$\beta_{v\times D}$		0.01	0.06	0.06		-0.01	0.01	0.03
		(0.06)	(0.06)	(0.06)		(0.07)	(0.07)	(0.06)
β_D			0.93***	0.93***			0.61	0.59*
			(0.35)	(0.35)			(0.41)	(0.36)
$\beta_{\pi_{-1}}$			0.28***	0.28***				0.44***
			(0.06)	(0.06)				(0.05)
N	256	256	255	255	256	256	256	255
adj. R^2	0.79	0.80	0.82	0.82	0.75	0.75	0.75	0.82

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for $\log\theta_t > 0$ and v is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 1960Q1-2023Q4.

Table F.4: Estimation of Equation (3), IV, Long Sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{π^e}	1.08***	1.04***	1.06***	0.57***	0.97***	0.99***	1.00***	0.32***
	(0.05)	(0.04)	(0.04)	(0.18)	(0.06)	(0.07)	(0.07)	(0.08)
β_θ	0.31	0.19	-0.07	0.02	0.38	0.38	0.24	0.25*
	(0.27)	(0.25)	(0.28)	(0.16)	(0.31)	(0.33)	(0.38)	(0.14)
$\beta_{\theta\times D}$	3.71***	4.01***	2.23**	0.34	-0.90	-1.15	-2.20*	-2.44***
	(0.94)	(0.90)	(1.10)	(1.13)	(1.25)	(1.27)	(1.13)	(0.65)
β_D			1.08*	0.83**			0.56	0.60*
			(0.59)	(0.42)			(0.40)	(0.31)
β_v		0.05**	0.05**	0.03		-0.01	-0.01	-0.00
		(0.03)	(0.03)	(0.02)		(0.03)	(0.03)	(0.02)
$\beta_{v\times D}$		0.02	0.06	0.05		-0.00	0.02	0.04
		(0.07)	(0.07)	(0.05)		(0.09)	(0.08)	(0.05)
$\beta_{\pi_{-1}}$				0.47***				0.71***
				(0.17)				(0.08)
Observations	254	254	254	253	254	254	254	253

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{D\times\theta} D_t \times \log\theta_t + \beta_v v_t + \beta_{D\times v} D_t \times v_t + \beta_{\pi_{-1}}\pi_{t-1} + \beta_D D_t + \varepsilon_t$*. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for* $\log\theta_t > 0$ *and* v *is a measure of supply shocks. Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* $\log\theta_t$ *and* π_{t-1} *are instrumented by their two first lags. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 1960Q1-2023Q4.*

G Aggregate Estimation Results Using a Cubic Specification

Here we estimate a more flexible form of nonlinearity using the cubic specification:

$$\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t \tag{G.1}$$

In the following tables, we find that with the experts measure of expectations (Columns (1) to (3)), the coefficient on $(\log\theta_t)^3$ (β_{θ^3}) is positive and significant. Nevertheless, when the household measure of expectation is chosen (Columns (4) to (6)), that coefficient becomes insignificant. Furthermore, the whole Phillips curve is estimated to be flat.

Table G.5: Estimation of Equation (G.1), OLS, Post Volcker

	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	0.86***	0.86***	0.64***	0.95***	1.09***	0.69***
	(0.08)	(0.08)	(0.11)	(0.13)	(0.13)	(0.14)
β_θ	1.83***	1.83***	1.31***	0.64**	0.49	0.14
	(0.24)	(0.24)	(0.29)	(0.31)	(0.31)	(0.28)
β_{θ^2}	2.69***	2.76***	1.90***	0.47	0.59	-0.02
	(0.46)	(0.48)	(0.54)	(0.60)	(0.59)	(0.54)
β_{θ^3}	1.08***	1.12***	0.77***	0.36	0.50	0.14
	(0.25)	(0.27)	(0.28)	(0.32)	(0.31)	(0.29)
β_v		-0.01	-0.01		-0.05***	-0.03**
		(0.01)	(0.01)		(0.02)	(0.02)
$\beta_{\pi_{-1}}$			0.26***			0.42***
			(0.08)			(0.07)
N	144	144	144	144	144	144
adj. R^2	0.61	0.61	0.63	0.48	0.51	0.61

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$*, where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 1988Q1-2023Q4.*

Table G.6: Estimation of Equation (G.1), IV, Post Volcker

	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	0.91***	0.90***	0.38	0.86***	0.96***	0.21*
	(0.10)	(0.09)	(0.32)	(0.26)	(0.24)	(0.11)
β_θ	1.61***	1.62***	0.47	0.75*	0.66*	-0.15
	(0.20)	(0.21)	(0.53)	(0.40)	(0.39)	(0.17)
β_{θ^2}	2.79***	2.90***	1.01	0.57	0.81	-0.43
	(0.39)	(0.39)	(1.23)	(0.84)	(0.80)	(0.46)
β_{θ^3}	1.17***	1.24***	0.44	0.36	0.56	-0.16
	(0.22)	(0.22)	(0.57)	(0.45)	(0.42)	(0.27)
β_v		-0.01	-0.01		-0.05***	-0.01
		(0.02)	(0.01)		(0.02)	(0.01)
$\beta_{\pi_{-1}}$			0.61*			0.88***
			(0.32)			(0.08)
Observations	144	144	144	144	144	144

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$, *where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* $\log\theta_t$, $(\log\theta_t)^2$, $(\log\theta_t)^3$ *and* π_{t-1} *are instrumented by their two first lags. Sample is 1988Q1-2023Q4.*

Table G.7: Estimation of Equation (G.1), OLS, Post 2007

	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	0.64**	0.66**	0.66**	0.89***	0.99***	1.10***
	(0.26)	(0.26)	(0.27)	(0.19)	(0.20)	(0.21)
β_θ	1.89***	1.86***	1.79***	1.56***	1.42***	1.83***
	(0.44)	(0.45)	(0.59)	(0.38)	(0.38)	(0.47)
β_{θ^2}	2.30***	2.44***	2.36**	0.05	0.25	0.63
	(0.76)	(0.82)	(0.96)	(0.89)	(0.88)	(0.91)
β_{θ^3}	0.91**	1.01**	0.98**	-0.24	-0.07	0.04
	(0.40)	(0.45)	(0.49)	(0.45)	(0.46)	(0.46)
β_v		-0.01	-0.01		-0.03	-0.04*
		(0.02)	(0.02)		(0.02)	(0.02)
$\beta_{\pi_{-1}}$			0.02			-0.18
			(0.14)			(0.13)
N	64	64	64	64	64	64
adj. R^2	0.59	0.59	0.58	0.67	0.68	0.69

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$*, where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 2008Q1-2023Q4.*

Table G.8: Estimation of Equation (G.1), IV Using Monetary Shock as an Instrument, Post 2007

	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	1.10**	0.99*	1.00*	1.01***	1.00***	1.18***
	(0.44)	(0.59)	(0.59)	(0.28)	(0.32)	(0.39)
β_{θ}	2.78	2.75	2.84	2.65*	1.94	3.21*
	(2.33)	(2.22)	(2.41)	(1.42)	(1.28)	(1.76)
β_{θ^2}	4.03	5.38*	5.49*	1.55	2.88	5.07
	(2.76)	(3.13)	(3.16)	(2.01)	(2.79)	(3.36)
β_{θ^3}	1.54	2.44*	2.47*	0.20	1.32	2.22
	(0.95)	(1.36)	(1.38)	(0.79)	(1.35)	(1.57)
β_{v}		-0.05	-0.05		-0.07**	-0.10***
		(0.04)	(0.04)		(0.03)	(0.03)
$\beta_{\pi_{-1}}$			-0.02			-0.27
			(0.25)			(0.21)
Observations	64	64	64	64	64	64

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$*, where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* π^e_{t+1}, $\log\theta_t$ *and* π_{t-1} *are instrumented by six lags of a measure of monetary shocks and six lags of their square. The measure of monetary shocks is the updated series of Bu, Rogers, and Wu [2021], as kindly provided by the authors. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 2008Q1-2023Q4.*

Table G.9: Estimation of Equation (G.1), OLS, Long Sample

	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	1.09***	1.03***	0.72***	0.97***	0.97***	0.58***
	(0.04)	(0.04)	(0.07)	(0.04)	(0.04)	(0.06)
β_θ	1.87***	1.88***	1.31***	-0.33	-0.33	-0.23
	(0.30)	(0.29)	(0.30)	(0.32)	(0.32)	(0.28)
β_{θ^2}	2.67***	2.56***	1.40**	0.01	0.00	-0.56
	(0.60)	(0.58)	(0.59)	(0.64)	(0.64)	(0.56)
β_{θ^3}	1.12***	1.01***	0.51	0.36	0.36	-0.06
	(0.33)	(0.32)	(0.31)	(0.35)	(0.35)	(0.31)
β_v		0.05***	0.03***		-0.00	-0.00
		(0.01)	(0.01)		(0.01)	(0.01)
$\beta_{\pi_{-1}}$			0.31***			0.43***
			(0.06)			(0.05)
N	256	256	255	256	256	255
adj. R^2	0.78	0.80	0.82	0.76	0.76	0.82

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$*, where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). Sample is 1960Q1-2023Q4.*

Table G.10: Estimation of Equation (G.1), IV, Long Sample

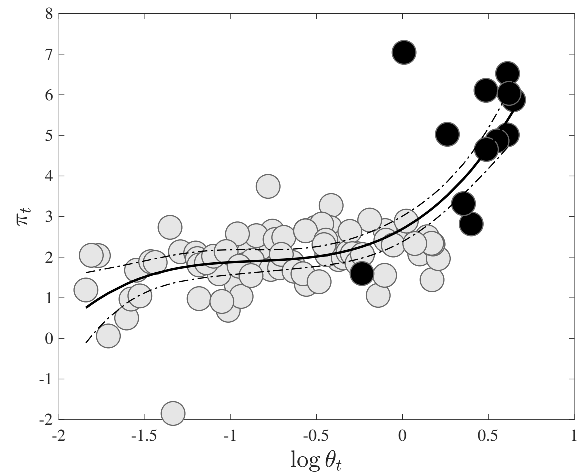
	(1)	(2)	(3)	(4)	(5)	(6)
β_{π^e}	1.08***	1.03***	0.50***	0.96***	1.00***	0.31***
	(0.05)	(0.04)	(0.16)	(0.05)	(0.06)	(0.08)
β_{θ}	1.87***	1.82***	0.91***	-0.29	-0.39	-0.03
	(0.28)	(0.29)	(0.29)	(0.45)	(0.44)	(0.20)
β_{θ^2}	2.38***	2.37***	0.64	-0.13	-0.29	-0.69*
	(0.61)	(0.60)	(0.58)	(0.83)	(0.82)	(0.37)
β_{θ^3}	0.95***	0.95***	0.12	0.25	0.21	-0.33
	(0.33)	(0.35)	(0.31)	(0.43)	(0.43)	(0.22)
β_v		0.05*	0.02		-0.02	0.00
		(0.02)	(0.02)		(0.03)	(0.02)
$\beta_{\pi_{-1}}$			0.52***			0.71***
			(0.15)			(0.08)
Observations	254	254	253	254	254	253

Notes: The generic Phillips curve we estimate is $\pi_t = \beta_{\pi^e}\pi^e_{t+1} + \beta_\theta \log\theta_t + \beta_{\theta^2}(\log\theta_t)^2 + \beta_{\theta^3}(\log\theta_t)^3 + \beta_v v_t + \beta_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$*, where* v *is a measure of supply shocks. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) to (4) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (5) to (8) use the Michigan Survey of Consumers measure of inflation expectations. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D).* $\log\theta_t$ *and* π_{t-1} *are instrumented by their two first lags. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 1960Q1-2023Q4.*

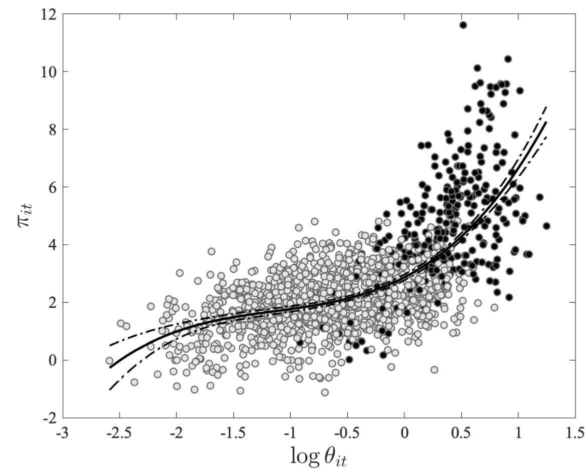
H Extra Figures

Figure H.1: Labour Market Tightness and Inflation, Raw Data

(a) Aggregate data

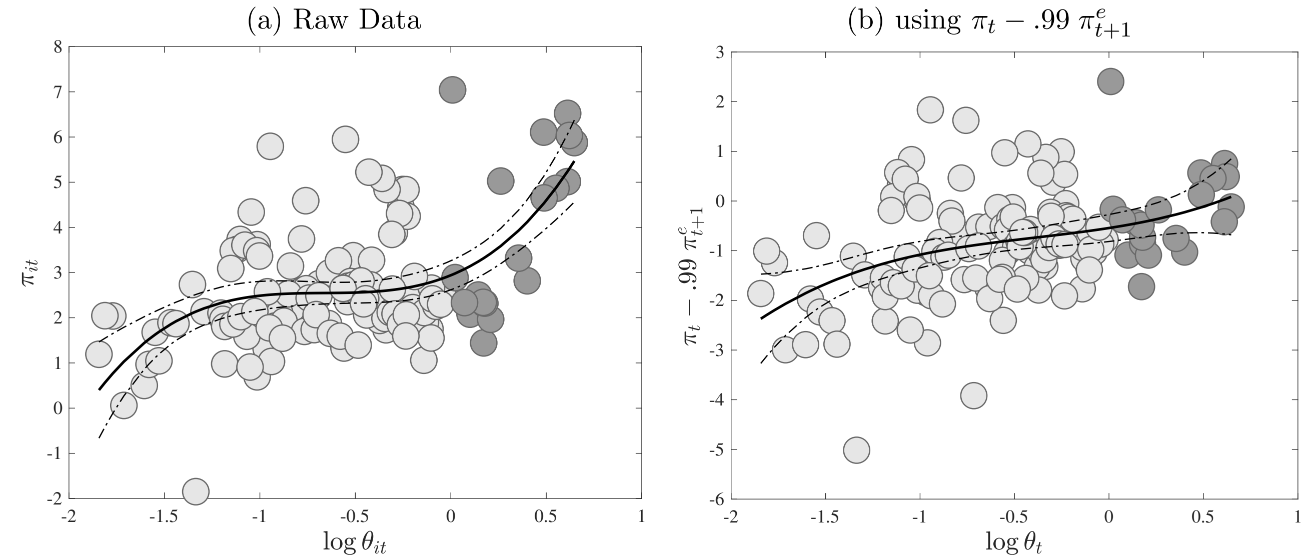


(b) City level data



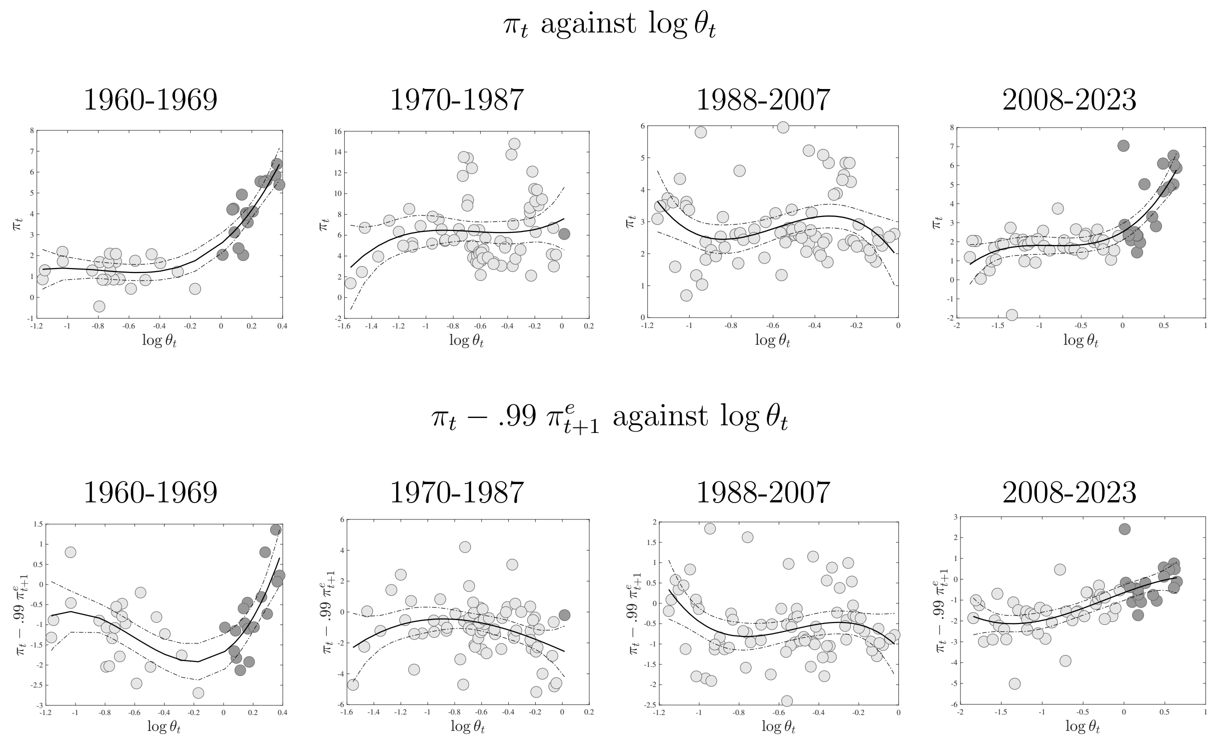
Notes: Labour market tightness is measured as $\log \theta$*, where* $\theta = V/U$*. Inflation is quarter-to-quarter CPI core inflation (annualized). Light gray dots correspond to quarters before 2021Q1, black dots correspond to quarters after 2021Q1 . The black line is the fitted cubic relation between* π *and* $\log \theta$*, dotted lines delimit the 95% confidence interval. Sample is 2000Q1-2023Q4 for Panel (a) and 200Q3-2024Q3 for Panel (b).*

Figure H.2: Inflation and Labour Market Tightness with Expectations, Aggregate Data, Post Volcker



Notes: Each dot represents a quarter (Panel (a)) or a quarter-city (Panel (b)). Dark dots indicate observations with $\log \theta_{it} \geq 0$ and light dots observations with $\log \theta_{it} < 0$. Inflation is quarter-to-quarter CPI core inflation (annualized). We use inflation expectations from the Michigan Survey of Consumers for expectations. It is measured as one year-ahead and adjusted to obtain a one quarter-ahead expectation (see Appendix D). The black line is the fitted cubic relation between the y-axis variable and $\log \theta$, dotted lines delimit the 95% confidence interval. Sample is 1988Q1-2023Q4.

Figure H.3: Inflation and Labour Market Tightness, Various Subperiods



Notes: Each dot represents a quarter. Dark dots indicate observations with $\log \theta_{it} \geq 0$ *and light dots observations with* $\log \theta_{it} < 0$. *Inflation is quarter-to-quarter CPI core inflation (annualized). We use median inflation expectations from the Michigan Survey of Consumers for the subperiods 1960-1969 and 1970-1987 as median ones are not available before 1978. Both measures are one year-ahead and adjusted to obtain one quarter-ahead expectation (see Appendix D). The black line is the fitted cubic relation between the y-axis variable and* $\log \theta$, *dotted lines delimit the 95% confidence interval.*